
$\operatorname{CF}\left[\mathbb{E}\left[\omega_{-}, L_{-}, Q_{-}, P_{-}\right]\right]:=$Expand /@ Together /@

## Utilities

 $\mathbb{E}\left[\omega / . \mathbf{b}_{L_{-}}: \rightarrow \log \left[\mathrm{t}_{l}\right], L, Q / . \mathrm{b}_{L_{-}}: \rightarrow \log \left[\mathrm{t}_{l}\right]\right.$, $\left.P / . b_{L_{-}} \rightarrow \log \left[t_{l}\right]\right] ;$$\mathbb{E} /: \mathbb{E}\left[\omega 1_{-}, L 1_{-}, Q 1_{-}, P 1_{-}\right] \mathbb{E}\left[\omega 2_{-}, L 2_{-}, Q 2_{-}, P 2_{-}\right]:=$ $\mathrm{CF} @ \mathbb{E}\left[\omega 1 \omega 2, L 1+L 2, \omega 2 Q 1+\omega 1 Q 2, \omega 2^{4} P 1+\omega 1^{4} P 2\right]$;

Normal Ordering Operators
$\mathbf{N}_{\mathrm{u}_{-}} \mathrm{c}_{j_{-} \rightarrow k_{-}}\left[\mathbb{E}\left[\omega_{-}, L_{-}, Q_{-}, P_{-}\right]\right]:=\operatorname{With}\left[\left\{q=e^{-\gamma} \beta \mathrm{u}_{k}+\gamma \mathrm{c}_{k}\right\}, \mathrm{CF}[\right.$
$\mathbb{E}\left[\omega, \gamma c_{k}+\left(L /, c_{j} \rightarrow 0\right)\right.$, d $\mathbb{e}^{-\gamma} \beta \mathbf{u}_{k}+\left(Q / . \mathbf{u}_{i} \rightarrow 0\right)$,

$$
\left.\left.\left.\mathbf{e}^{-q} \mathbf{D P}_{\mathrm{c}_{j} \rightarrow \mathrm{D}_{\gamma}, u_{i} \rightarrow \mathrm{D}_{\beta}}[P]\left[\mathrm{e}^{q}\right]\right] / .\left\{\gamma \rightarrow \partial_{\mathrm{c}_{j}} L, \beta \rightarrow \boldsymbol{L}^{-1} \partial_{u_{i}} Q\right\}\right]\right] ;
$$

$\mathbf{N}_{w_{i_{-}}} c_{j_{-} \rightarrow k_{-}}\left[\mathbb{E}\left[\omega_{-}, L_{-}, Q_{-}, P_{-}\right]\right]:=\operatorname{With}\left[\left\{q=e^{\gamma} \alpha w_{k}+\gamma c_{k}\right\}, C F[\right.$
$N_{w_{i_{-}}} u_{j_{-} \rightarrow k_{-}}\left[\mathbb{E}\left[\omega_{-}, L_{-}, Q_{-}, P_{-}\right]\right]:=$

$$
\text { With }\left[\left\{q=\left(1-t_{k}\right) \mu^{-1} \alpha \frac{\beta}{W}+\mu^{-1} \beta u_{k}+\mu^{-1} \delta u_{k} w_{k}+\mu^{-1} \alpha w_{k}\right\}, \text { CF }[\right.
$$

$$
\mathbb{E}\left[\mu \nless_{2} L, \mu \nless \mathrm{q}+\mu\left(Q / \cdot \mathrm{w}_{i} \mid \mathrm{u}_{j} \rightarrow 0\right) / \mathbb{W}\right.
$$

$$
\left.W^{-4} \mu^{4} e^{-q} \mathrm{DP}_{\mathrm{w}_{i} \rightarrow \mathrm{D}_{\alpha}, u_{j} \rightarrow \mathrm{D}_{\beta}}[P]\left[\mathrm{e}^{\mathrm{q}}\right]+\omega^{4} \Lambda[k]\right] /
$$

$$
\mu \rightarrow(\mathbb{X})+\left(t_{k}-1\right) \delta /
$$

$$
\left\{\alpha \rightarrow \omega^{-2}\left(\partial_{w_{i}} Q / \cdot u_{j} \rightarrow 0\right), \beta \rightarrow \omega \chi^{2 z}\left(\partial_{u_{j}} Q / \cdot w_{i} \rightarrow 0\right),\right.
$$

$$
\left.\left.\left.\delta \rightarrow \omega \chi^{z} \partial_{\mathrm{w}_{i}}, \mathrm{u}_{j} Q\right\}\right]\right] ;
$$

$\Lambda\left[k_{-}\right]:=\left(1-t_{k}\right)\left(\alpha^{2} \beta^{2}+4 \alpha \beta / \delta \mu+2 \delta^{2} V^{2}\right) / 2+2 \mu^{2}(\alpha \beta+\delta \mu) c_{k}-$ $\beta(2 \mu-1)(\alpha \beta+2 \delta \mu) u_{k}+2 \beta \delta \mu^{2} c_{k} u_{k}-\beta^{2} \delta(3 \mu-1) u_{k}^{2} / 2+$ $\alpha(\alpha \beta+2 \delta \mu) \mathbf{w}_{k}+2 \alpha \delta \mu^{2} c_{k} \mathbf{w}_{k}-2\left(\mathbf{t}_{k}-1\right) \delta^{2}(\alpha \beta+\delta \mu) \mathbf{u}_{k} \mathbf{w}_{k}+$ $2 \delta^{2} \mu^{2} c_{k} u_{k} \mathbf{w}_{k}-\beta \delta^{2}(2 \mu-1) u_{k}^{2} w_{k}+\alpha^{2} \delta(1+\mu) w_{k}^{2} / 2+$ $\alpha \delta^{2} u_{k} w_{k}^{2}-\left(t_{k}-1\right) \delta^{4} u_{k}^{2} w_{k}^{2} / 2$;

The $\Lambda$ ó $\gamma$ os

$$
\begin{aligned}
& \mathbb{E}\left[\omega, \gamma c_{k}+\left(L /, c_{j} \rightarrow 0\right), \gamma \mathbb{e}^{\gamma} \alpha w_{k}+\left(Q / \cdot w_{i} \rightarrow 0\right)\right. \text {, } \\
& \left.\left.\left.\mathbb{e}^{-q} \mathrm{DP}_{\mathrm{c}_{j} \rightarrow \mathrm{D}_{\gamma}, \mathrm{w}_{i} \rightarrow \mathrm{D}_{\alpha}}[P]\left[\mathrm{e}^{q}\right]\right] / .\left\{\gamma \rightarrow \partial_{\mathrm{c}_{j}} L, \alpha \rightarrow \omega \chi^{z} \partial_{\mathrm{w}_{i}} Q\right\}\right]\right] ;
\end{aligned}
$$

