2-Cocycles: Local to Global

April 8, 2016 7:15 PM

R: a rep of a Lie algebra $\mathfrak{g}$

$$0 \longrightarrow R \longrightarrow \mathfrak{g} \ltimes R \underset{\ell}{\overset{u}{\leftrightarrows}} \mathfrak{g} \longrightarrow 0$$

The difference between two splittings is a 1-cocycle.

2-cocycles: $\rho : \mathfrak{g} \otimes \mathfrak{g} \longrightarrow R$

$$[(x,r),(x',r')] := ([x,x'],\ x\cdot r' - x' r + \rho(x,x'))$$

ρ should be a 2-cocycle: AS+

$$[[x,y],z] \longrightarrow -z\cdot\rho(x,y) + \rho([x,y],z)$$

so $\rho([x,y],z) - z\cdot\rho(x,y) + \text{cyc perms} = 0.$

Q: Is there a group 2-cocycle for every Lie algebra 2-cocycle?

Group 2-cocycles: $1 \longrightarrow A \longrightarrow G \ltimes A \longrightarrow G \longrightarrow 1$

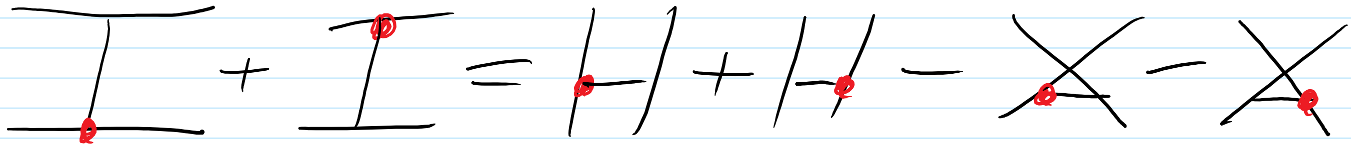
2-cocycle: $\alpha : G \times G \longrightarrow A$

$$(g,a)\cdot(g',a') = (gg',\ a' + a \triangleleft g' + \alpha(g,g'))$$

α should be a group 2-cocycle:

$$x(yz) = (xy)z \Rightarrow \alpha(y,z) + \alpha(x,yz) = \alpha(x,y) \triangleleft z + \alpha(xy,z)$$

Algebra 2-cocycles, diagrammatically: ● : ρ



General formulas seem hopeless. Perhaps something may work in near-Abelian situations.
My Lie algebra is not near Abelian!
But it is "near $gl(N)$".