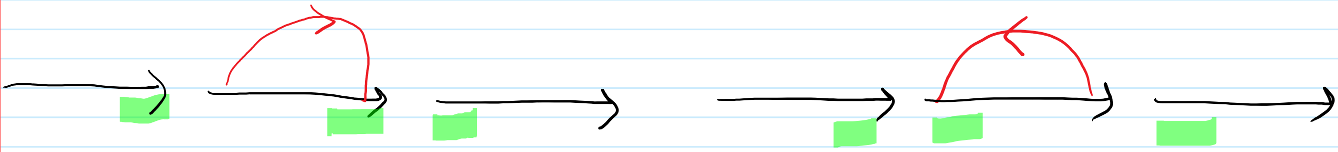
Gauss-Gassner-Alexander

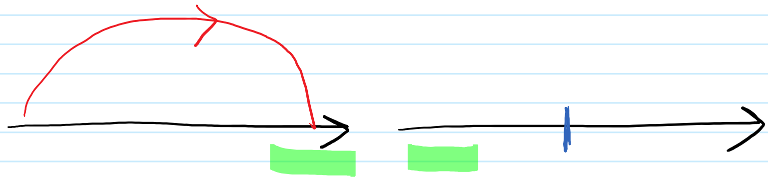
March 23, 2016 11:48 AM



: target regions.

Is there a "rolling map", to reduce things to 2x2 matrices?

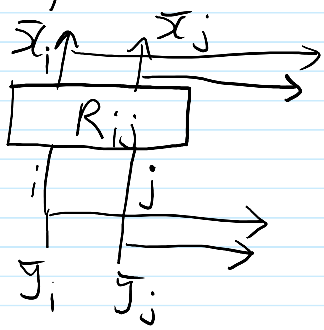
The 2x2 way:



Deriving Gassner. $\mathcal{L}^{2Dw}$ is $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo locality, $[a_{ij},a_{ik}]=0$, $[a_{ik},a_{jk}]=-[a_{ij},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, and (mod $\langle a_{ii}\rangle$) $[a_{ij},a_{ji}]=b_ia_{ji}-b_ja_{ij}$. Acts on $V=\mathbb{Q}[\![b_i]\!]\langle x_i=a_{i\infty}\rangle$ by $[a_{ij},x_i]=0$, $[a_{ij},x_j]=b_ix_j-b_jx_i$. Hence $e^{\mathrm{ad}\,a_{ij}}x_i=x_i$, $e^{\mathrm{ad}\,a_{ij}}x_j=e^{b_i}x_j+\frac{b_j}{b_i}(1-e^{b_i})x_i$. Renaming $\bar{x}_i=x_i/b_i$, $t_i=e^{b_i}$, get $[e^{\mathrm{ad}\,a_{ij}}]_{\bar{x}_i,\bar{x}_j}=\begin{pmatrix}1&1-t_i\\0&t_i\end{pmatrix}$.

$$Rp_{a\_,b\_} := r\left[1,\ \mathrm{Tr}\left[\begin{pmatrix}t_a\\t_b\end{pmatrix}^{T}.\begin{pmatrix}1&1-T_a\\0&T_a\end{pmatrix}.\begin{pmatrix}h_a\\h_b\end{pmatrix}\right]\right];$$

That is,

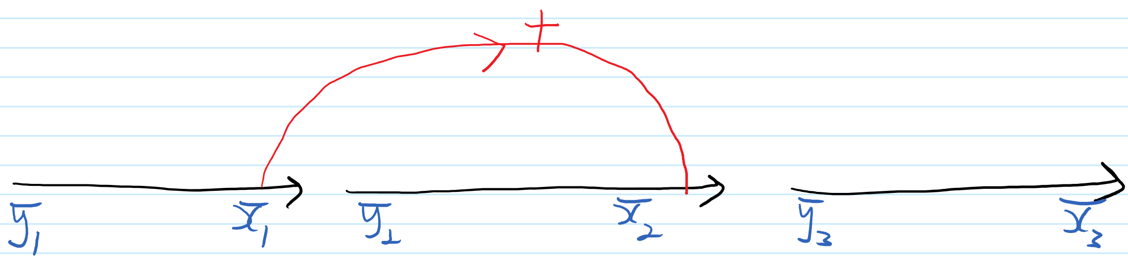


$$\bar{x}_i = \bar{y}_i$$

$$\bar{x}_j = (1-t_i)\bar{y}_i + t_i\bar{y}_j$$

or

$$(\bar{x}_i\ \ \bar{x}_j) = (\bar{y}_i\ \ \bar{y}_j)\begin{pmatrix}1&1-t_i\\0&t_i\end{pmatrix}$$



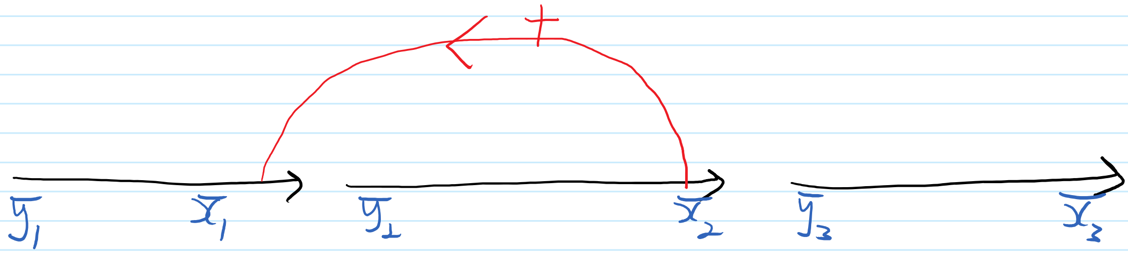
$$A=\begin{pmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{pmatrix}$$

$$\bar{x}_1 = a_{11}\bar{y}_1 + a_{21}\bar{y}_2 + a_{31}\bar{y}_3$$

$$\bar{x}_2 = a_{12}\bar{y}_1 + a_{22}\bar{y}_2 + a_{32}\bar{y}_3$$

$$\bar{x}_3 = a_{13}\bar{y}_1 + a_{23}\bar{y}_2 + a_{33}\bar{y}_3$$

$\bar{x}_1 = \bar{y}_2 \quad \bar{x}_2 = 0 \quad \bar{y}_3 = 1$ (tentatively)



$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

$$\bar{x}_1 = a_{11}\bar{y}_1 + a_{21}\bar{y}_2 + a_{31}\bar{y}_3$$
$$\bar{x}_2 = a_{12}\bar{y}_1 + a_{22}\bar{y}_2 + a_{32}\bar{y}_3$$
$$\bar{x}_3 = a_{13}\bar{y}_1 + a_{23}\bar{y}_2 + a_{33}\bar{y}_3$$

$\bar{x}_2 = \bar{y}_3 \quad \bar{x}_1 = 0 \quad \bar{y}_2 = 1$ (tentatively)

```
In[15]:= Simplify[x1 /. Solve[{
           x1 == a11 y1 + a21 y2 + a31 y3,
           x2 == a12 y1 + a22 y2 + a32 y3,
           x3 == a13 y1 + a23 y2 + a33 y3,
           x1 == y2, x2 == 0, y3 == 1},
          {x1, x2, x3, y1, y2, y3}]]
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Out[15]= $\left\{\dfrac{a12\ a31 - a11\ a32}{a12 - a12\ a21 + a11\ a22}\right\}$

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In[16]:= Simplify[x2 /. Solve[{
           x1 == a11 y1 + a21 y2 + a31 y3,
           x2 == a12 y1 + a22 y2 + a32 y3,
           x3 == a13 y1 + a23 y2 + a33 y3,
           x2 == y3, x1 == 0, y2 == 1},
          {x1, x2, x3, y1, y2, y3}]]
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Out[16]= $\left\{\dfrac{-a12\ a21 + a11\ a22}{a11 + a12\ a31 - a11\ a32}\right\}$