The Abstract Context, 2

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The Abstract Context. (From LesDiablerets-1508)

Definition. A meta-monoid is a functor M: (finite sets, injections)→(sets) along with natural operations $*\colon M(S_1)\times M(S_2)\to M(S_1\sqcup S_2)$ whenever $S_1\cap S_2=\emptyset$ and $m^{ab}_c\colon M(S)\to M((S\setminus\{a,b\})\sqcup\{c\})$ whenever $a\neq b\in S$ and $c\notin S\setminus\{a,b\}$, such that

$$\text{meta-associativity:}\quad m^{ab}_y /\!/ m^{yc}_x = m^{bc}_y /\!/ m^{ay}_x$$

$$\text{meta-locality:}\quad m^{ab}_x /\!/ m^{de}_y = m^{de}_y /\!/ m^{ab}_x$$

and, with $\epsilon_b = M(S\hookrightarrow S\sqcup\{b\})$,

$$\text{meta-unit:}\quad \epsilon_b /\!/ m^{ab}_a = Id = \epsilon_b /\!/ m^{ba}_a.$$

Theorem. $S\mapsto\Gamma_0(S)$ is a meta-monoid and $z_0\colon \mathit{PT}\to\Gamma_0$ is a morphism of meta-monoids.

Theorem. There exists an extension of Γ_0 to a bigger meta-monoid $\Gamma_{01}(S)=\Gamma_0(S)\times\Gamma_1(S)$, along with an extension of z_0 to $z_{01}\colon \mathit{PT}\to\Gamma_{01}$, with

$$\Gamma_1(S)=R_S\oplus V\oplus V^{\otimes 2}\oplus V^{\otimes 3}\oplus \mathcal{S}^2(V)^{\otimes 2}\qquad (\text{with } V:=R_S\langle S\rangle).$$

Furthermore, upon reducing to a single variable everything is polynomial size and polynomial time.

Furthermore, Γ_{01} is given using a "meta-2-cocycle ρ^{ab}_c over Γ_0": In addition to $m^{ab}_c\to m^{ab}_{0c}$, there are R_S-linear $m^{ab}_{1c}\colon\Gamma_1(S\sqcup\{a,b\})\to\Gamma_1(S\sqcup\{c\})$, a meta-right-action $\alpha^{ab}\colon\Gamma_1(S)\times\Gamma_0(S)\to\Gamma_1(S)$ R_S-linear in the first variable, and a first order differential operator (over R_S) $\rho^{ab}_c\colon\Gamma_0(S\sqcup\{a,b\})\to\Gamma_1(S\sqcup\{c\})$ such that

$$(\zeta_0,\zeta_1)/\!/m^{ab}_c=\left(\zeta_0/\!/m^{ab}_{0c},\,(\zeta_1,\zeta_0)/\!/\alpha^{ab}/\!/m^{ab}_{1c}+\zeta_0/\!/\rho^{ab}_c\right)$$

Perhaps I need to construct a "Contraction Machine".