

The Abstract Context, 1

January 18, 2016 10:09 AM

The Abstract Context. (From LesDiablerets-1508)

Definition. A meta-monoid is a functor $M: (\text{finite sets, injections}) \rightarrow (\text{sets})$ along with natural operations $*$: $M(S_1) \times M(S_2) \rightarrow M(S_1 \sqcup S_2)$ whenever $S_1 \cap S_2 = \emptyset$ and $m_c^{ab}: M(S) \rightarrow M((S \setminus \{a, b\}) \sqcup \{c\})$ whenever $a \neq b \in S$ and $c \notin S \setminus \{a, b\}$, such that

$$\text{meta-associativity: } m_{xy}^{ab} // m_{bc}^{yc} = m_{yz}^{bc} // m_{ax}^{ay}$$

$$\text{meta-locality: } m_{xc}^{ab} // m_{yd}^{de} = m_{yc}^{de} // m_{xc}^{ab}$$

and, with $\epsilon_b = M(S \hookrightarrow S \sqcup \{b\})$,

$$\text{meta-unit: } \epsilon_b // m_a^{ab} = Id = \epsilon_b // m_a^{ba}.$$

~~Claim. Pure virtual tangles PVT form a meta-monoid.~~

Theorem. $S \mapsto \Gamma_0(S)$ is a meta-monoid and $z_0: PVT \rightarrow \Gamma_0$ is a morphism of meta-monoids.

~~Strong Conviction.~~ There exists an extension of Γ_0 to a bigger meta-monoid $\Gamma_{01}(S) = \Gamma_0(S) \times \Gamma_1(S)$, along with an extension of z_0 to $z_{01}: PVT \rightarrow \Gamma_{01}$, with

$$\Gamma_1(S) = R_S \oplus V \oplus V^{\otimes 2} \oplus V^{\otimes 3} \oplus S^2(V)^{\otimes 2} \quad (\text{with } V := R_S \langle S \rangle).$$

Furthermore, upon reducing to a single variable everything is polynomial size and polynomial time.

Furthermore, Γ_{01} is given using a “meta-2-cocycle ρ_c^{ab} over Γ_0 ”: In addition to $m_c^{ab} \rightarrow m_{0c}^{ab}$, there are R_S -linear $m_{1c}^{ab}: \Gamma_1(S \sqcup \{a, b\}) \rightarrow \Gamma_1(S \sqcup \{c\})$, a meta-right-action $\alpha^{ab}: \Gamma_1(S) \times \Gamma_0(S) \rightarrow \Gamma_1(S)$ R_S -linear in the first variable, and a first order differential operator (over R_S) $\rho_c^{ab}: \Gamma_0(S \sqcup \{a, b\}) \rightarrow \Gamma_1(S \sqcup \{c\})$ such that

$$(\zeta_0, \zeta_1) // m_c^{ab} = (\zeta_0 // m_{0c}^{ab}, (\zeta_1, \zeta_0) // \alpha^{ab} // m_{1c}^{ab} + \zeta_0 // \rho_c^{ab})$$

~~What's done? The braid part, with still ugly formulas.~~

~~What's missing? A lot of concept and detail-sensitive work towards m_{1c}^{ab} , α^{ab} , and ρ_c^{ab} . The “ribbon element”.~~

Theorem.