Lukic: KdV equation with almost periodic initial data

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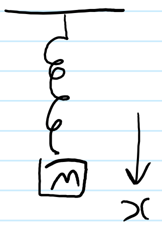
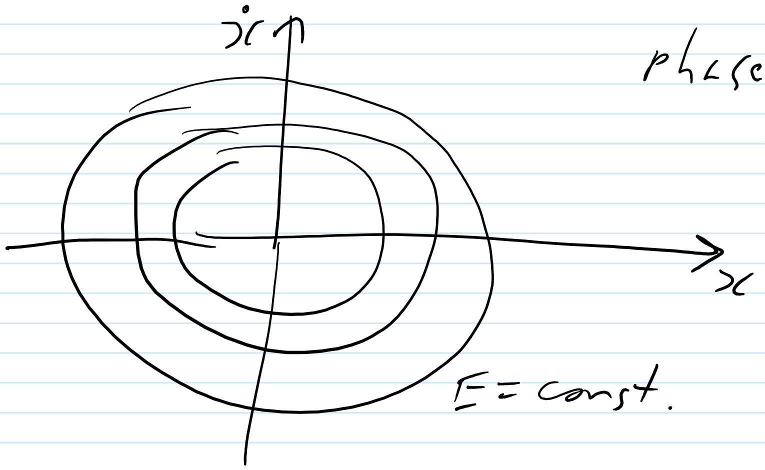


$m\ddot{x} = -kx$

$E = m\dot{x}^2/2 + kx^2/2 = \text{const}$



phase space is 2D

$\begin{pmatrix} x \\ \dot{x} \end{pmatrix} = M(\omega t)$

$M : \mathbb{T} \to \mathbb{R}^2$

$\omega \in \mathbb{R}$

We seek an ∞-dim analog, for the KdV equation.

$$\partial_t u - 6u\partial_x u + \partial_x^3 u = 0 \qquad t \in \mathbb{R}$$

$$u(x,0) = V(x) \qquad x \in \mathbb{R} \text{ or } \mathbb{T}$$

write the Schrödinger operator

$$H(t) = -\partial_x^2 + u(x,t) \quad \text{on} \quad L^2(\mathbb{R})$$

$H(t)$ evolves with t. Set

$$B(t) = 4\partial_x^3 u + 3(\partial_x u + u\partial_x)$$

then $\partial_t H(t) = [B(t), H(t)]$ "Lax pair"

Consider the unitary operators given by

$$\partial_t U(t) = B(t)U(t) \qquad U(0) = I$$

then

$V(t)^* H(t) U(t) = H(0)$ (by differentiating lhs & getting 0)

So $H(t)$ is unitarily equiv. to $H(0)$. 0:15

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under some bounds, in the periodic case in x, can find $M : \mathbb{T}^J \longrightarrow H^n(\mathbb{T})$

Sobolev space

s.t. $u(\cdot, t) = M(\xi t)$ w/ $\xi \in \mathbb{R}^J$

here J is the set of open gaps in the spectrum of $H(0)$.

Def'n: $F : \mathbb{R} \longrightarrow \mathcal{B}$ is $\mathcal{B}$-almost periodic if $F(t) = M(\xi t)$, w/ $M : \mathbb{T}^\infty \longrightarrow \mathcal{B}$ ....

<u>Conj (Deift):</u> If $V(x)$ is $\mathbb{R}$-almost periodic the $u(\cdot, t)$ is almost periodic in t.

<u>Thm</u> (Binder–Damanik–Goldstein–L)