A pairing-expansion example

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$$\langle t^n, t^m \rangle = \frac{(t^n-1)(t^m-1)}{t-1} \qquad n,m \geq 1$$

$$\langle x^n, x^m \rangle = \begin{cases} 0 & (n=0) \vee (m=0) \\ x^{n+m-1} & \text{otherwise.} \end{cases}$$

Is there a homomorphic expansion?

Try $t \longrightarrow e^x$

$$\langle t^n, t^m \rangle \longrightarrow \frac{(e^{nx}-1)(e^{mx}-1)}{e^x-1}$$

$$\downarrow$$

$$\langle e^{nx}, e^{mx} \rangle = \langle e^{nx}-1, e^{mx}-1 \rangle =$$

$$= \frac{(e^{nx}-1)(e^{mx}-1)}{x}$$

$\Longrightarrow$

Perhaps there should be a "pairing vertex" whose value is $\frac{e^x-1}{x}$?