

Pensieve header: Demos for my Qinhuangdao-1507 talk.

The .nb (source) file is at <http://drorbn.net/AcademicPensieve/2015-07/PolyPoly/>.

```
SetDirectory["C:/drorbn/AcademicPensieve/2015-07/PolyPoly/"]
```

```
C:\drorbn\AcademicPensieve\2015-07\PolyPoly
```

Initialization

```
ΓCollect[Γ[ω_, λ_] := Γ[Simplify[ω],
  Collect[λ, h_, Collect[#, t_, Factor] &]];
Format[Γ[ω_, λ_] := Module[{S, M},
  S = Union@Cases[Γ[ω, λ], (h | t)_a_ -> a, ∞];
  M = Outer[Factor[∂_{h_{#1} t_{#2}} λ] &, S, S];
  M = Prepend[M, t_# & /@ S] // Transpose;
  M = Prepend[M, Prepend[h_# & /@ S, ω]];
  M // MatrixForm];
```

Program

```
Γ /: Γ[ω1_, λ1_] Γ[ω2_, λ2_] := Γ[ω1 * ω2, λ1 + λ2];
m_{a_b -> c}_[Γ[ω_, λ_]] := Module[{α, β, γ, δ, θ, ε, φ, ψ, Ξ, μ},
  (α β θ
   γ δ ε
   φ ψ Ξ) = (∂_{t_a, h_a} λ ∂_{t_a, h_b} λ ∂_{t_a} λ
             ∂_{t_b, h_a} λ ∂_{t_b, h_b} λ ∂_{t_b} λ
             ∂_{h_a} λ ∂_{h_b} λ λ) /. (t | h)_{a|b} -> 0;
  Γ[(μ = 1 - β) ω, {t_c, 1} . (γ + α δ / μ ε + δ θ / μ
                                φ + α ψ / μ Ξ + ψ θ / μ) . {h_c, 1}]
  /. {T_a -> T_c, T_b -> T_c} // ΓCollect];
Rp_{a_b} := Γ[1, {t_a, t_b} . (1 1 - T_a
                               0 T_a) . {h_a, h_b}];
Rm_{a_b} := Rp_{ab} /. T_a -> 1 / T_a;
```

MetaAssoc

```
ξ = Γ[ω, {t_1, t_2, t_3, t_s} . (α_{11} α_{12} α_{13} θ_1
                                   α_{21} α_{22} α_{23} θ_2
                                   α_{31} α_{32} α_{33} θ_3
                                   φ_1 φ_2 φ_3 Ξ) . {h_1, h_2, h_3, h_s}];
(ξ // m_{12 -> 1} // m_{13 -> 1}) == (ξ // m_{23 -> 2} // m_{12 -> 1})
```

MetaAssoc

True

R3

```
{Rm_{51} Rm_{62} Rp_{34} // m_{14 -> 1} // m_{25 -> 2} // m_{36 -> 3},
 Rp_{61} Rm_{24} Rm_{35} // m_{14 -> 1} // m_{25 -> 2} // m_{36 -> 3}}
```

R3

$$\left\{ \begin{pmatrix} 1 & h_1 & h_2 & h_3 \\ t_1 & \frac{T_3}{T_2} & 0 & 0 \\ t_2 & \frac{-1+T_2}{T_2} & \frac{1}{T_3} & 0 \\ t_3 & -\frac{-1+T_3}{T_2} & \frac{-1+T_3}{T_3} & 1 \end{pmatrix}, \begin{pmatrix} 1 & h_1 & h_2 & h_3 \\ t_1 & \frac{T_3}{T_2} & 0 & 0 \\ t_2 & \frac{-1+T_2}{T_2} & \frac{1}{T_3} & 0 \\ t_3 & -\frac{-1+T_3}{T_2} & \frac{-1+T_3}{T_3} & 1 \end{pmatrix} \right\}$$

8\_17

$\mathbf{z} = \mathbf{Rm}_{12,1} \mathbf{Rm}_{27} \mathbf{Rm}_{83} \mathbf{Rm}_{4,11} \mathbf{Rp}_{16,5} \mathbf{Rp}_{6,13} \mathbf{Rp}_{14,9} \mathbf{Rp}_{10,15};$

$\mathbf{Do}[\mathbf{z} = \mathbf{z} // \mathbf{m}_{1 \rightarrow 1}, \{\mathbf{k}, 2, 16\}];$

$\mathbf{z}$

8\_17

$$\begin{pmatrix} 11 - \frac{1}{T_1^2} + \frac{4}{T_1^2} - \frac{8}{T_1} - 8 T_1 + 4 T_1^2 - T_1^3 & h_1 \\ & 1 \end{pmatrix}$$

tr

$\mathbf{tr}_c[\Gamma[\omega, \lambda]] := \mathbf{Module}[\{\alpha, \theta, \psi, \Xi\},$   
 $\begin{pmatrix} \alpha & \theta \\ \psi & \Xi \end{pmatrix} = \begin{pmatrix} \partial_{t_c, h_c} \lambda & \partial_{t_c} \lambda \\ \partial_{h_c} \lambda & \lambda \end{pmatrix} /. (\mathbf{t} | \mathbf{h})_c \rightarrow 0;$   
 $\Gamma[\omega (1 - \alpha), \Xi + \psi * \theta / (1 - \alpha)] // \mathbf{RCollect};$   
 $(\xi // \mathbf{m}_{12 \rightarrow 1} // \mathbf{tr}_1) = (\xi // \mathbf{m}_{21 \rightarrow 1} // \mathbf{tr}_1)$

tr

True

## Extras

### Meta-Associativity in detail

$\mathbf{Column}@\{\xi // \mathbf{m}_{12 \rightarrow 1}, \xi // \mathbf{m}_{23 \rightarrow 2}, \xi // \mathbf{m}_{12 \rightarrow 1} // \mathbf{m}_{13 \rightarrow 1}, \xi // \mathbf{m}_{23 \rightarrow 2} // \mathbf{m}_{12 \rightarrow 1}\}$

$$\begin{pmatrix} \omega - \omega \alpha_{12} & h_1 & h_3 & h_s \\ t_1 & \frac{-\alpha_{21} + \alpha_{12} \alpha_{21} - \alpha_{11} \alpha_{22}}{-1 + \alpha_{12}} & \frac{-\alpha_{13} \alpha_{22} - \alpha_{23} + \alpha_{12} \alpha_{23}}{-1 + \alpha_{12}} & \frac{-\alpha_{22} \theta_1 - \theta_2 + \alpha_{12} \theta_2}{-1 + \alpha_{12}} \\ t_3 & \frac{-\alpha_{31} + \alpha_{12} \alpha_{31} - \alpha_{11} \alpha_{32}}{-1 + \alpha_{12}} & \frac{-\alpha_{13} \alpha_{32} - \alpha_{33} + \alpha_{12} \alpha_{33}}{-1 + \alpha_{12}} & \frac{-\alpha_{32} \theta_1 - \theta_3 + \alpha_{12} \theta_3}{-1 + \alpha_{12}} \\ t_s & \frac{-\phi_1 + \alpha_{12} \phi_1 - \alpha_{11} \phi_2}{-1 + \alpha_{12}} & \frac{-\alpha_{13} \phi_2 - \phi_3 + \alpha_{12} \phi_3}{-1 + \alpha_{12}} & \frac{-\Xi + \Xi \alpha_{12} - \theta_1 \phi_2}{-1 + \alpha_{12}} \end{pmatrix}$$

$$\begin{pmatrix} \omega - \omega \alpha_{23} & h_1 & h_2 & h_s \\ t_1 & \frac{-\alpha_{11} - \alpha_{13} \alpha_{21} + \alpha_{11} \alpha_{23}}{-1 + \alpha_{23}} & \frac{-\alpha_{12} - \alpha_{13} \alpha_{22} + \alpha_{12} \alpha_{23}}{-1 + \alpha_{23}} & \frac{-\theta_1 + \alpha_{23} \theta_1 - \alpha_{13} \theta_2}{-1 + \alpha_{23}} \\ t_2 & \frac{-\alpha_{31} + \alpha_{23} \alpha_{31} - \alpha_{21} \alpha_{33}}{-1 + \alpha_{23}} & \frac{-\alpha_{32} + \alpha_{23} \alpha_{32} - \alpha_{22} \alpha_{33}}{-1 + \alpha_{23}} & \frac{-\alpha_{33} \theta_2 - \theta_3 + \alpha_{23} \theta_3}{-1 + \alpha_{23}} \\ t_s & \frac{-\phi_1 + \alpha_{23} \phi_1 - \alpha_{21} \phi_3}{-1 + \alpha_{23}} & \frac{-\phi_2 + \alpha_{23} \phi_2 - \alpha_{22} \phi_3}{-1 + \alpha_{23}} & \frac{-\Xi + \Xi \alpha_{23} - \theta_2 \phi_3}{-1 + \alpha_{23}} \end{pmatrix}$$

$$\begin{pmatrix} \omega (1 - \alpha_{13} \alpha_{22} + \alpha_{12} (-1 + \alpha_{23}) - \alpha_{23}) & h_1 \\ t_1 & \frac{\alpha_{31} - \alpha_{12} \alpha_{31} - \alpha_{13} \alpha_{22} \alpha_{31} - \alpha_{23} \alpha_{31} + \alpha_{12} \alpha_{23} \alpha_{31} + \alpha_{11} \alpha_{32} \alpha_{23} \alpha_{31} + \alpha_{11} \alpha_{32} + \alpha_{13} \alpha_{21} \alpha_{32} - \alpha_{11} \alpha_{23} \alpha_{32} + \alpha_{21} \alpha_{33} - \alpha_{12} \alpha_{21}}{1 - \alpha_{12} - \alpha_{13} \alpha_{22} - \alpha_{23} + \alpha_{12} \alpha_{23}} \\ t_s & \frac{\phi_1 - \alpha_{12} \phi_1 - \alpha_{13} \alpha_{22} \phi_1 - \alpha_{23} \phi_1 + \alpha_{12} \alpha_{23} \phi_1 + \alpha_{11} \phi_2 + \alpha_{13} \alpha_{21} \phi_2 - \alpha_{11} \alpha_{23} \phi_2 + \alpha_{21} \phi_3 - \alpha_{12} \alpha_{21} \phi_3}{1 - \alpha_{12} - \alpha_{13} \alpha_{22} - \alpha_{23} + \alpha_{12} \alpha_{23}} \end{pmatrix}$$

$$\begin{pmatrix} \omega (1 - \alpha_{13} \alpha_{22} + \alpha_{12} (-1 + \alpha_{23}) - \alpha_{23}) & h_1 \\ t_1 & \frac{\alpha_{31} - \alpha_{12} \alpha_{31} - \alpha_{13} \alpha_{22} \alpha_{31} - \alpha_{23} \alpha_{31} + \alpha_{12} \alpha_{23} \alpha_{31} + \alpha_{11} \alpha_{32} + \alpha_{13} \alpha_{21} \alpha_{32} - \alpha_{11} \alpha_{23} \alpha_{32} + \alpha_{21} \alpha_{33} - \alpha_{12} \alpha_{21}}{1 - \alpha_{12} - \alpha_{13} \alpha_{22} - \alpha_{23} + \alpha_{12} \alpha_{23}} \\ t_s & \frac{\phi_1 - \alpha_{12} \phi_1 - \alpha_{13} \alpha_{22} \phi_1 - \alpha_{23} \phi_1 + \alpha_{12} \alpha_{23} \phi_1 + \alpha_{11} \phi_2 + \alpha_{13} \alpha_{21} \phi_2 - \alpha_{11} \alpha_{23} \phi_2 + \alpha_{21} \phi_3 - \alpha_{12} \alpha_{21} \phi_3}{1 - \alpha_{12} - \alpha_{13} \alpha_{22} - \alpha_{23} + \alpha_{12} \alpha_{23}} \end{pmatrix}$$

### Four types of R1

$\{\mathbf{Rp}_{12} // \mathbf{m}_{12 \rightarrow 1}, \mathbf{Rp}_{12} // \mathbf{m}_{21 \rightarrow 1}, \mathbf{Rm}_{12} // \mathbf{m}_{12 \rightarrow 1}, \mathbf{Rm}_{12} // \mathbf{m}_{21 \rightarrow 1}\}$

$$\left\{ \begin{pmatrix} T_1 & h_1 \\ t_1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & h_1 \\ t_1 & 1 \end{pmatrix}, \begin{pmatrix} \frac{1}{T_1} & h_1 \\ t_1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & h_1 \\ t_1 & 1 \end{pmatrix} \right\}$$

## Two types of R2

$\{\mathbf{Rp}_{12} \mathbf{Rm}_{34} // \mathbf{m}_{13 \rightarrow 1} // \mathbf{m}_{24 \rightarrow 2}, \mathbf{Rp}_{12} \mathbf{Rm}_{34} // \mathbf{m}_{13 \rightarrow 1} // \mathbf{m}_{42 \rightarrow 2}\}$

$$\left\{ \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & h_1 & h_2 \\ t_1 & 1 & 0 \\ t_2 & 0 & 1 \end{pmatrix} \right\}$$