The Av/2D Lie algebra

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2Dv. b: bracket trace; c: cobracket trace; $\langle b, c\rangle = \delta \in \{0,1\}$. Relations: locality, $[a_{ij}, a_{ik}] = c_k a_{ij} - c_j a_{ik}$, $[a_{ik}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, $[a_{ij}, a_{jk}] = (c_j - b_j)a_{ik} + b_i a_{jk} - c_k a_{ij}$, $[a_{ij}, a_{ji}] = ?$, $[a_{ij}, b_i] = -[a_{ij}, b_j] = -[a_{ij}, c_i] = [a_{ij}, c_j] = \delta a_{ij} - b_i c_j$, $[b_i, c_i] = 0$.

Problem. Figure out the bracket

$$[f + f^{ij} a_{ij}, g + g^{ij} a_{ij}]$$

Enough to understand $[a_{ij}, g]$. (Warmup bellow)

Guess based on Warmup:

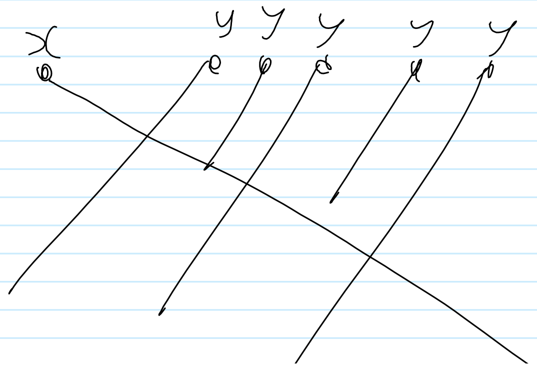
$$a_{ij} f = \left(f^\delta - \frac{b_i c_j}{\delta}(f^\delta - f)\right) a_{ij}$$

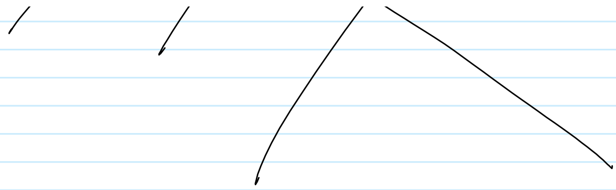
with $f^\delta = f\Big|_{\substack{b_i \to b_i + \delta \quad c_i \to c_i - \delta \\ b_j \to b_j - \delta \quad c_j \to c_j + \delta}}$

so $[a_{ij}, f] = \left(1 - \frac{b_i c_j}{\delta}\right)(f^\delta - f) a_{ij}$

Warmup: Compute $[x, f(y)]$ under $[x, y] = \delta x$

Sol'n





$$x f(y) = f(y+\delta) x$$

Indeed, $$x y^n = x y y^{n-1} = (yx + \delta x) y^{n-1}$$
$$= (y+\delta) x y^{n-1} = (y+\delta)(y+\delta)^{n-1} x$$
$$= (y+\delta)^n x$$