OneCo-150418

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Cheat Sheet OneCo

http://drorbn.net/AcademicPensieve/2015-04/
initiated 14/4/15; modified 18/4/15, 2:40pm

Background. $\delta e^\gamma = e^\gamma \cdot \left(\delta\gamma /\!\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma}\right) = \left(\delta\gamma /\!\!/ \frac{e^{\operatorname{ad}\gamma}-1}{\operatorname{ad}\gamma}\right)\cdot e^\gamma$

The differential of $\gamma = \operatorname{bch}(\alpha,\beta)$:

$$\delta\gamma /\!\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} = \left(\delta\alpha /\!\!/ \frac{1-e^{-\operatorname{ad}\alpha}}{\operatorname{ad}\alpha} /\!\!/ e^{-\operatorname{ad}\beta}\right) + \left(\delta\beta /\!\!/ \frac{1-e^{-\operatorname{ad}\beta}}{\operatorname{ad}\beta}\right)$$

Models. • In $[x,y]=\delta x$, $xf(y)=f(y+\delta)x$. If $\delta^2=0$, $[x,f(y)]=\delta f'(y)x$.
• In $[x,y]=\delta x+z^2$, $xf(y)=f(y+\delta)x+\frac{z^2}{\delta}(f(y+\delta)-f(y))$. If $\delta^2=0$, $[x,f(y)]=\delta f'(y)x+z^2f'(y)$.

Deriving Gassner. $\mathcal{L}^w$ is $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo locality, $[a_{ij},a_{ik}]=0$, $[a_{ik},a_{jk}]=-[a_{ij},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, and $[a_{ij},a_{ji}]=b_ia_{ji}-b_ja_{ij}$. Acts on $V=\mathbb{Q}[\![b_i]\!]\langle x_i=a_{i\infty}\rangle$ by $[a_{ij},x_i]=0$, $[a_{ij},x_j]=b_ix_j-b_jx_i$. Hence $e^{\operatorname{ad}a_{ij}}x_i=x_i$, $e^{\operatorname{ad}a_{ij}}x_j=e^{b_i}x_j+\frac{b_j}{b_i}(1-e^{b_i})x_i$.

Renaming $y_i=x_i/b_i$, $t_i=e^{b_i}$, get $[e^{\operatorname{ad}a_{ij}}]_{y_i,y_j}=\begin{pmatrix}1 & 1-t_i\\ 0 & t_i\end{pmatrix}$.

The $\mathcal{L}^w$ Adjoint representation. $e^{\operatorname{ad}a_{ij}}$ acts by

$$a_{kl}\mapsto a_{kl},\qquad a_{ik}\mapsto a_{ik},\qquad a_{kj}\mapsto e^{-b_i}a_{kj}+\frac{b_k}{b_i}(1-e^{-b_i})a_{ij},$$

$$\text{and}\quad a_{jk}\mapsto e^{b_i}a_{jk}+\frac{b_j}{b_i}(1-e^{b_i})a_{ik}\qquad \text{also for } k=i.$$

An interpretation?

2Dv. b: bracket trace; c: cobracket trace; $\langle b,c\rangle=\delta\in\{0,1\}$; $\deg b_i=\deg c_j=\deg a_{ij}=\deg\delta=1$.

$\mathcal{L}^v$ is $\mathbb{Q}[\![\delta]\!]\left\langle\mathbb{Q}[\![b_i,c_j]\!]\langle 1,a_{ij}\rangle\right\rangle$ (so $\mathcal{L}^v=\{f+f^{ij}a_{ij}\}$) modulo locality, $[a_{ij},a_{ik}]=c_ka_{ij}-c_ja_{ik}$, $[a_{ik},a_{jk}]=b_ja_{ik}-b_ia_{jk}$, $[a_{ij},a_{jk}]=(c_j-b_j)a_{ik}+b_ia_{jk}-c_ka_{ij}$, $[a_{ij},a_{ji}]=?$, and $[a_{ij},b_i]=-[a_{ij},b_j]=-[a_{ij},c_i]=[a_{ij},c_j]=\delta a_{ij}-b_ic_j$.

$$a_{ij}f=f^\delta a_{ij}-\frac{b_ic_j}{\delta}(f^\delta-f),\quad [a_{ij},f]=(f^\delta-f)\left(a_{ij}-\frac{b_ic_j}{\delta}\right),$$

with $f^\delta := f /\!\!/ \begin{pmatrix} b_i\to b_i+\delta & b_j\to b_j-\delta\\ c_i\to c_i-\delta & c_j\to c_j+\delta\end{pmatrix}$.

The primitivity condition. $\ker\left(f+f^{ij}a_{ij}\mapsto \delta f+f^{ij}b_ic_j\right)$. $\mathcal{L}^v_+$

The OneCo Quotient is $\delta c_i=c_jc_k=0$, so $\mathcal{L}^{1co}=\left\{(f+f^kc_k)+(f^{ij}+f^{ijk}c_k)a_{ij}\colon f,f^i,f^{ij},f^{ijk}\in\mathbb{Q}[\![\delta,b_i]\!]\right\}$. Then $[a_{ij},f+f^kc_k]=(\delta a_{ij}-b_ic_j)(\partial_if-\partial_jf-f^i+f^j)$.

Next. $e^{\operatorname{ad}a_{ij}}(f)=?$

To do. • Perhaps I should find a way to highlight the fact that v is a perturbation of w.

$e^{\operatorname{ad}a_{ij}}$:

$$\begin{tikzcd}
\{f c_j\} \arrow[r, "\hat b_i"] \arrow[d, "-"'] & \{f c_j\} \arrow[loop right, "-\hat b_i"] \arrow[dl, "+"'] \\
\{a_{ij} f\} \arrow[d] & \{f\} \arrow[u, "\hat b_i(\partial_j-\partial_i)"'] \arrow[l, "\partial_i-\partial_j"] \\
0 &
\end{tikzcd}$$

$$\begin{tikzcd}
a_{ik} \arrow[r, "+"] \arrow[d, "-"'] & c_k a_{ij} \arrow[d] \\
c_j a_{ik} \arrow[r] & 0
\end{tikzcd}$$

$[a_{ij},a_{kj}]=b_ka_{ij}-b_ia_{kj}$ so

$$\begin{tikzcd}
a_{kj} \arrow[loop left, "-\hat b_i"] \arrow[r, "\hat b_k"] & a_{ij} \arrow[r] & 0
\end{tikzcd}$$

$$\begin{tikzcd}
a_{jk} \arrow[loop left, "\hat b_i"] \arrow[r, "+"] \arrow[dr, "-"] \arrow[dd, "-\hat b_j"'] & c_j a_{ik} \arrow[loop above, "-\hat b_i"] \arrow[r, "+"] & \delta a_{ij} a_{jk} \arrow[dl] \\
& c_k a_{ij} \arrow[r] & 0 \\
a_{ik} \arrow[ur, "+"] \arrow[r, "-"] & c_j a_{ik} \arrow[ur] &
\end{tikzcd}$$

In[1]:= `MatrixExp[(a c / 0 b)] // MatrixForm`

Out[1]//MatrixForm=

$$\begin{pmatrix} e^a & \frac{c\,(e^a-e^b)}{a-b}\\ 0 & e^b\end{pmatrix}$$

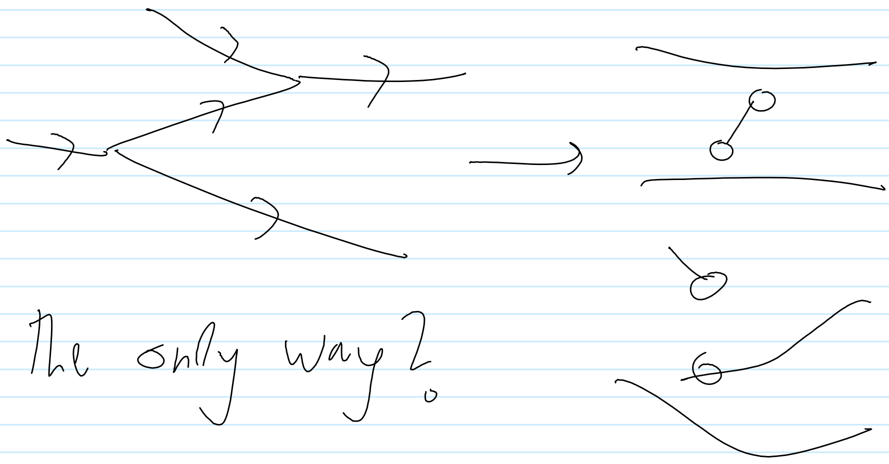
$$S_n := \sum_{k=0}^{n-1} A^k C B^{n-k}$$

$$AS_n - S_nB = A^nC - CB^n$$

$$(L_A-R_B)S_n = A^nC-CB^n$$

$$S_n = (L_A-R_B)^{-1}(A^nC-CB^n)$$

$a_{ij}a_{ik}$ dah? Can this arise in a β-γ reduction of a connected 1-co diagram?



etc.

is this the only way?

YES.