OneCo-150417

April-17-15 9:24 AM

A: $\deg b = \deg c_j = \deg a_{ij} = \deg \delta = 1$

Cheat Sheet OneCo

http://drorbn.net/AcademicPensieve/2015-04/
initiated 14/4/15; modified 17/4/15, 9:47am

Background. $\delta e^\gamma = e^\gamma \cdot \left(\delta\gamma /\!\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma}\right) = \left(\delta\gamma /\!\!/ \frac{e^{\operatorname{ad}\gamma}-1}{\operatorname{ad}\gamma}\right)\cdot e^\gamma$

The differential of $\gamma = \operatorname{bch}(\alpha,\beta)$:

$$\delta\gamma /\!\!/ \frac{1-e^{-\operatorname{ad}\gamma}}{\operatorname{ad}\gamma} = \left(\delta\alpha /\!\!/ \frac{1-e^{-\operatorname{ad}\alpha}}{\operatorname{ad}\alpha} /\!\!/ e^{-\operatorname{ad}\beta}\right) + \left(\delta\beta /\!\!/ \frac{1-e^{-\operatorname{ad}\beta}}{\operatorname{ad}\beta}\right)$$

Models. • In $[x,y] = \delta x$, $xf(y) = f(y+\delta)x$. If $\delta^2 = 0$, $[x, f(y)] = \delta f'(y)$.

add $[x,y] = \delta x + y^2$

B

Deriving Gassner. $\mathcal{L}^w$ is $\mathbb{Q}[\![b_i]\!]\langle a_{ij}\rangle$ modulo locality, $[a_{ij}, a_{ik}] = 0$, $[a_{ik}, a_{jk}] = -[a_{ij}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, and $[a_{ij}, a_{ji}] = b_i a_{ji} - b_j a_{ij}$. Acts on $V = \mathbb{Q}[\![b_i]\!]\langle x_i = a_{i\infty}\rangle$ by $[a_{ij}, x_i] = 0$, $[a_{ij}, x_j] = b_i x_j - b_j x_i$. Hence $e^{\operatorname{ad} a_{ij}} x_i = x_i$, $e^{\operatorname{ad} a_{ij}} x_j = e^{b_i} x_j + \frac{b_j}{b_i}(1-e^{b_i})x_i$.

Renaming $y_i = x_i/b_i$, $t_i = e^{b_i}$, get $[e^{\operatorname{ad} a_{ij}}]_{y_i,y_j} = \begin{pmatrix}1 & 1-t_i\\ 0 & t_i\end{pmatrix}$.

The $\mathcal{L}^w$ Adjoint representation. $e^{\operatorname{ad} a_{ij}}$ acts by

$$a_{kl} \mapsto a_{kl}, \qquad a_{ik} \mapsto a_{ik}, \qquad a_{kj} \mapsto e^{-b_i} a_{kj} + \frac{b_k}{b_i}(1-e^{-b_i})a_{ij},$$

and $\quad a_{jk} \mapsto e^{b_i} a_{jk} + \frac{b_j}{b_i}(1-e^{b_i})a_{ik} \qquad$ also for $k = i$.

An interpretation?

2Dv. b: bracket trace; c: cobracket trace; $\langle b, c\rangle = \delta \in \{0,1\}$. A

$\mathcal{L}^v$ is $\mathbb{Q}[\![\delta]\!]\left\langle \mathbb{Q}[\![b_i, c_j]\!]\langle 1, a_{ij}\rangle\right\rangle$ (so $\mathcal{L}^v = \{f + f^{ij}a_{ij}\}$) modulo locality, $[a_{ij}, a_{ik}] = c_k a_{ij} - c_j a_{ik}$, $[a_{ik}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, $[a_{ij}, a_{jk}] = (c_j - b_j)a_{ik} + b_i a_{jk} - c_k a_{ij}$, $[a_{ij}, a_{ji}] =$?, and $[a_{ij}, b_i] = -[a_{ij}, b_j] = -[a_{ij}, c_i] = [a_{ij}, c_j] = \delta a_{ij} - b_i c_j$.

$a_{ij} f = \left(f^\delta - \cancel{\frac{b_i c_j}{\delta}}(f^\delta - f)\right)a_{ij}$, $\quad [a_{ij}, f] = \left(1 - \cancel{\frac{b_i c_j}{\delta}}\right)(f^\delta - f)a_{ij}$,

with $f^\delta := f /\!\!/ \begin{pmatrix} b_i\to b_i+\delta & b_j \to b_j - \delta \\ c_i \to c_i - \delta & c_j \to c_j + \delta\end{pmatrix}$.

The primitivity condition. $\ker\left(f + f^{ij}a_{ij} \mapsto \delta f + f^{ij} b_i c_j\right)$.

The OneCo Quotient is $\delta c_i = c_j c_k = 0$, so $\mathcal{L}^{1co} = \left\{(f + f^i c_i) + (f^{ij} + f^{ijk} c_k)a_{ij} \colon f, f^i, f^{ij}, f^{ijk} \in \mathbb{Q}[\![\delta, b_i]\!]\right\}$. Then $[a_{ij}, f + f^i c_i] = (\delta - b_i c_j)(\partial_{b_i} f - \partial_{b_j} f - f^i + f^j)a_{ij}$.

$i \quad j$

inconsistent Einstein summation.

degree mismatch!

Next: $e^{\operatorname{ad} a_{ij}} f$

Encounters:

$a_{ij}, f_i, c_j, (\partial_i - \partial_j), b_i$

Model:

$a, c, f(x,y), \partial_x, x+y$

$[a, f] = (x+y) \ldots$

To do. • Perhaps I should find a way to highlight the fact that v is a perturbation of w.

B: If $[x,y] = \delta x + z^2$ then

$$x f(y) = f(y+\delta)x +$$

$$xy = (y+\delta)x + z^2$$

$$xy^2 = (y+\delta)xy + z^2y = (y+\delta)^2x + (y+\delta)z^2 + z^2y$$

$$xy^3 = (y+\delta)^3x + (y+\delta)^2z^2 + (y+\delta)z^2y + z^2y^2$$

$$xy^n = (y+\delta)^n x + z^2\frac{(y+\delta)^n - y^n}{y+\delta-y}$$

$$= (y+\delta)^n x + \frac{z^2}{\delta}\left((y+\delta)^n - y^n\right)$$

So $x f(y) = f(y+\delta)x + \frac{z^2}{\delta}\left(f(y+\delta) - f(y)\right)$