

OneCo-150416

April-16-15 10:12 AM

* one column ~~X~~

* perhaps I should find a way to highlight the fact that v is a perturbation of w . ✓

Cheat Sheet OneCo

http://drorbn.net/AcademicPensieve/2015-04/
initiated 14/4/15; modified 16/4/15, 10:09am

Background.

$$d \exp: \quad \delta e^\gamma = e^\gamma \cdot \left(\delta \gamma \parallel \frac{1 - e^{-\text{ad } \gamma}}{\text{ad } \gamma} \right) = \left(\delta \gamma \parallel \frac{e^{\text{ad } \gamma} - 1}{\text{ad } \gamma} \right) \cdot e^\gamma$$

The differential of $\gamma = \text{bch}(\alpha, \beta)$:

$$\delta \gamma \parallel \frac{1 - e^{-\text{ad } \gamma}}{\text{ad } \gamma} = \left(\delta \alpha \parallel \frac{1 - e^{-\text{ad } \alpha}}{\text{ad } \alpha} \parallel e^{-\text{ad } \beta} \right) + \left(\delta \beta \parallel \frac{1 - e^{-\text{ad } \beta}}{\text{ad } \beta} \right)$$

Models. • In $[x, y] = \delta x$, $xf(y) = f(y + \delta)x$. If $\delta^2 = 0$, $[x, f(y)] = \delta f'(y)$.

Deriving Gassner. Locality, $[a_{ij}, a_{ik}] = 0$, $[a_{ik}, a_{jk}] = -[a_{ij}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, $[a_{ij}, a_{ji}] = b_i a_{ji} - b_j a_{ij}$, b_i central. Acts on $V = \mathbb{Q}[[b_i]]\langle x_i = a_{i\infty} \rangle$ by $[a_{ij}, x_i] = 0$, $[a_{ij}, x_j] = b_i x_j - b_j x_i$.

Hence $e^{\text{ad } a_{ij}} x_i = x_i$, $e^{\text{ad } a_{ij}} x_j = e^{b_i} x_j + \frac{b_j}{b_i} (1 - e^{b_i}) x_i$. Renaming

$$y_i = x_i / b_i, \quad t_i = e^{b_i}, \quad \text{get } [e^{\text{ad } a_{ij}}]_{y_i, y_j} = \begin{pmatrix} 1 & 1 - t_i \\ 0 & t_i \end{pmatrix}.$$

The $\mathcal{A}^w / 2D$ Adjoint representation. $e^{\text{ad } a_{ij}}$ acts by

$$a_{kl} \mapsto a_{kl}, \quad a_{ik} \mapsto a_{ik}, \quad a_{kj} \mapsto e^{-b_i} a_{kj} + \frac{b_k}{b_i} (1 - e^{-b_i}) a_{ij},$$

$$\text{and } a_{jk} \mapsto e^{b_i} a_{jk} + \frac{b_j}{b_i} (1 - e^{b_i}) a_{ik} \quad \text{also for } k = i.$$

2Dv. b : bracket trace; c : cobarcket trace; $\langle b, c \rangle = \delta \in \{0, 1\}$. Relations: locality, $[a_{ij}, a_{ik}] = c_k a_{ij} - c_j a_{ik}$, $[a_{ik}, a_{jk}] = b_j a_{ik} - b_i a_{jk}$, $[a_{ij}, a_{jk}] = (c_j - b_j) a_{ik} + b_i a_{jk} - c_k a_{ij}$, $[a_{ij}, a_{ji}] = ?$, $[a_{ij}, b_i] = -[a_{ij}, b_j] = -[a_{ij}, c_i] = [a_{ij}, c_j] = \delta a_{ij} - b_i c_j$, $[b_i, c_i] = 0$.

$$a_{ij} f = \left(f^\delta - \frac{b_i c_j}{\delta} (f^\delta - f) \right) a_{ij}, \quad [a_{ij}, f] = \left(1 - \frac{b_i c_j}{\delta} \right) (f^\delta - f) a_{ij},$$

$$\text{with } f^\delta := f \parallel \begin{pmatrix} b_i \rightarrow b_i + \delta & b_j \rightarrow b_j - \delta \\ c_i \rightarrow c_i - \delta & c_j \rightarrow c_j + \delta \end{pmatrix}.$$

The primitivity condition. $\ker (f + f^{ij} a_{ij} \mapsto \delta f + f^{ij} b_i c_j)$.

The OneCo Quotient is $\delta c_i = c_j c_k = 0$. Then $[a_{ij}, f] = (\delta - b_i c_j) [(\partial_{b_i} - \partial_{b_j} - \partial_{c_i} + \partial_{c_j}) f]_{c_i=0} \cdot a_{ij}$.

name the algebra. ✓
Basis for 1-co. ✓