Huizenga@Colloq: Interpolation problems in algebraic geometry

January-05-15  2:09 PM

Lagrangian Interpolation

$p_1, \ldots, p_n \in \mathbb{C} \text{ "points" (distinct)}$
$q_1, \ldots, q_n \in \mathbb{C} \text{ "values"}$

$\Rightarrow \exists \Phi \text{ poly of deg } \leq n-1 \text{ s.t. } \Phi(p_i) = q_i$

Proof \(\text{ev}: S_{n-1} \rightarrow \mathbb{C}^n\)

$\Phi \rightarrow (\Phi(p_1) - \ldots - \Phi(p_n))$

is 1-1 hence onto

Multivariable interpolation:

$p_1, \ldots, p_n \in \mathbb{C}^r$

What is the rank of

$\text{ev}: S_m \rightarrow \mathbb{C}^n$

poly's deg \(\leq m\)

Equivalently, how many indep. conditions does vanishing at \(n\) impose on \(S_m\)?

Example \(n=3\) pts in plane

$r=2, m=1$

\(\exists \text{ rank drops if}\)

\(p\text{'s colinear}\)

Deduc a collection of pts "has interpolation"
in deg m if even has max rank.

Keep several pts $p_i...p_n$ have interpolation in deg m.

Pick $p_1$ arbitrarily.
$p_2$ chosen s.t. not all $F$ that vanish at $p_1$ also vanish at $p_2$.
Keep going ... until every poly that vanishes on $p_1...p_n$ is the zero poly. Last this no matter what choice.

(1) What happens at special config?
(2) What if we prescribe derivatives as well?
(3) Impose general zeros on sections of vector bundles.

Motivation: Bivariant geometry of Hilbert schemes of points in $P^2$. 

20 minutes.