

Es<-->El Algebraically

December-17-14 10:36 AM

From cheat sheet FreeLie:

5. ✓ Γ : With $\Gamma(t) \in FL(T)^T$ solving $\Gamma(0) = 0$, $\Gamma'(s) = \lambda // e^{-\partial_{s\lambda}} // \frac{\text{ad } \Gamma(s)}{e^{\text{ad } \Gamma(s)} - 1}$, $e^{-\partial_\lambda} = C^{\Gamma(1)}$
6. ✓ Λ : With $\Lambda(t) \in FL(T)^T$ solving $\Lambda(0) = 0$, $\Lambda'(s) = \lambda // e^{\partial_{\Lambda(s)}} // \frac{\text{ad}_{tb} \Lambda(s)}{e^{\text{ad}_{tb} \Lambda(s)} - 1}$, $e^{-\partial_{\Lambda(1)}} = C^\lambda$

From WK04.nb (but will move):

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 $\lambda2 = \langle 1 \rightarrow \text{RandomLieSeries}[\{1, 2\}], 2 \rightarrow \text{RandomLieSeries}[\{1, 2\}] \rangle$ 
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$$\left(1 \rightarrow \text{LS}\left[\overline{2}, 0, -\frac{1}{3} \overline{1 \overline{1} \overline{2}} - \overline{1 \overline{2} \overline{2}}, \frac{41}{24} \overline{1 \overline{1} \overline{1} \overline{2}} + \frac{19}{12} \overline{1 \overline{1} \overline{2} \overline{2}} + \frac{7}{12} \overline{\overline{1} \overline{2} \overline{2} \overline{2}}, \dots \right], 2 \rightarrow \text{LS}\left[2 \overline{1} + 2 \overline{2}, -2 \overline{1} \overline{2}, -\frac{1}{3} \overline{1 \overline{1} \overline{2}} + \frac{5}{6} \overline{1 \overline{2} \overline{2}}, \frac{29}{24} \overline{1 \overline{1} \overline{1} \overline{2}} - \frac{11}{8} \overline{1 \overline{1} \overline{2} \overline{2}} - \frac{17}{12} \overline{\overline{1} \overline{2} \overline{2} \overline{2}}, \dots \right] \right)$$

$$\left\{ \begin{aligned} \text{lhs} &= \lambda2 // \text{EulerE} // \text{adSeries}\left[\frac{e^{\text{ad}} - 1}{\text{ad}}, \lambda2 \right] // \text{RC}[-\lambda2], \\ \text{rhs} &= \Lambda[\lambda2] // \text{EulerE} // \text{adSeries}\left[\frac{e^{\text{ad}} - 1}{\text{ad}}, \Lambda[\lambda2], \text{tb} \right]; (\text{lhs} \equiv \text{rhs}) @ \{8\} \end{aligned} \right\}$$

$$\left\{ \begin{aligned} &\left(1 \rightarrow \text{LS}\left[\overline{2}, -2 \overline{1} \overline{2}, \overline{1 \overline{1} \overline{2}} - \overline{1 \overline{2} \overline{2}}, \frac{15}{2} \overline{1 \overline{1} \overline{1} \overline{2}} + \frac{20}{3} \overline{1 \overline{1} \overline{2} \overline{2}} - \frac{5}{6} \overline{\overline{1} \overline{2} \overline{2} \overline{2}}, \dots \right], \right. \\ &2 \rightarrow \text{LS}\left[2 \overline{1} + 2 \overline{2}, -6 \overline{1} \overline{2}, 5 \overline{1 \overline{1} \overline{2}} + \frac{11}{2} \overline{1 \overline{2} \overline{2}}, \right. \\ &\left. \left. \frac{3}{2} \overline{1 \overline{1} \overline{1} \overline{2}} - \frac{45}{2} \overline{1 \overline{1} \overline{2} \overline{2}} - \frac{49}{6} \overline{\overline{1} \overline{2} \overline{2} \overline{2}}, \dots \right] \right), \text{BS}[9 \text{True}, \dots] \end{aligned} \right\}$$

RHS at $\Lambda(\lambda_2) \rightarrow \lambda_2$, meaning

$$\lambda_2 // E // \frac{e^{\text{ad}_{tb} \lambda_2} - 1}{\text{ad}_{tb} \lambda_2}$$

is $Z^{-1} E Z$ (up to signs etc.)