Doodles handout on Nov 2, 2014

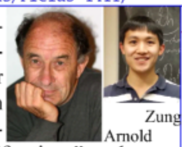
November-02-14 2:20 PM

Video, handout, links and more at ωεβ:=http://www.math.toronto.edu/~drorbn/Talks/Fields-1411/

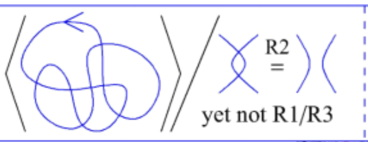
Dror Bar-Natan: Talks: Fields-1411:

Finite Type Invariants of Doodles, 1

Abstract. I will describe my former student's Jonathan Zung work on finite type invariants of "doodles", plane curves modulo the second Reidemeister move but not modulo the third. We use a definition of "finite type" different from Arnold's and more along the lines of Goussarov's "Interdependent Modifications", and come to a conjectural combinatorial description of the set of all such invariants. We then describe how to construct many such invariants (though perhaps not all) using a certain class of 2-dimensional "configuration space integrals". An unfinished project!

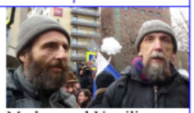

Zung Arnold

Doodles. $\mathcal{K} = \mathcal{K}_0 = \mathbb{Q}$

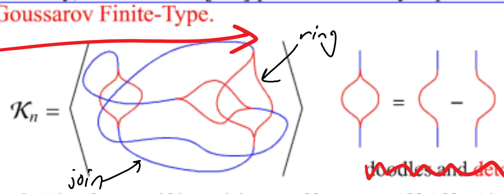


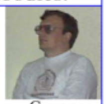
Easy to classify!

Prior Art. Arnold [Ar] first studied doodles within his study of plane curves and the "strangeness" St invariant. Vassiliev [Va1, Va2] defined finite type invariants in a different way, and Merkov [Me] proved that they separate doodles.


Merkov and Vassiliev

Goussarov Finite-Type.

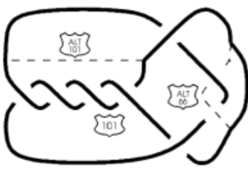


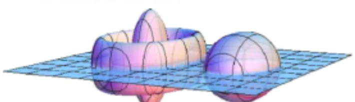

Goussarov

~~doodles and detours (dnd's)~~

Def. V is of type n if it vanishes on $\mathcal{K}_{n+1}$. $(\mathcal{K}_0/\mathcal{K}_{n+1})^\star \leftrightsquigarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

Knots in 3D. **2-Knots in 4D.**

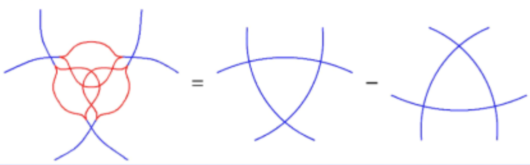




The reason I care!

Goals. • Describe $\mathcal{A}_n := \mathcal{K}_n/\mathcal{K}_{n+1}$ using diagrams/relations. • Get many or all finite type invariants of doodles using configurations space integrals. • Do these come from a TQFT? • See if $\mathcal{A}_n$ has a "Lie theoretic" (tensors/relations) meaning. • See if/how Arnold's St and the Merkov invariants integrate in.

Important Example.

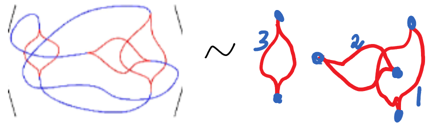


Summary Diagram. MC: (Multi-Commutator) relations.

$$\begin{tikzcd} & \mathcal{K} \arrow[d, Rightarrow] \arrow[dr, "Z"] & & \\ \mathcal{D}^c/MC =: \mathcal{A}^c \arrow[r, two heads, "\pi"] \arrow[rr, bend right, "T"'] & \mathrm{gr}\mathcal{K} \arrow[r, "\mathrm{gr}Z"] & \mathcal{A}^t := \mathcal{D}^t/FDR \arrow[r, two heads, "?"] & \mathcal{A}^c \end{tikzcd}$$

An Upper Bound on $\mathcal{K}_n/\mathcal{K}_{n+1}$.

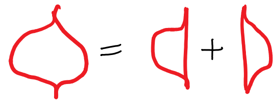
The Rayman Principle In $\mathcal{K}_n/\mathcal{K}_{n+1}$,



"joins are irrelevant"

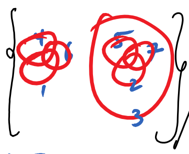

Rayman by Ubisoft

The subdivision relations



⟹ Rings can be subdivided until each participates in just one feature.

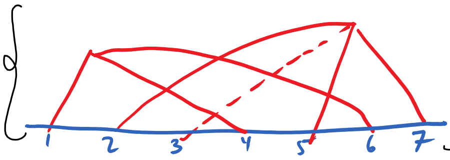
So...



AS
Tetrahedron
ring exchange

$\twoheadrightarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

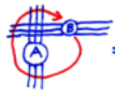
"chord Diagrams"



AS
Tet
RE

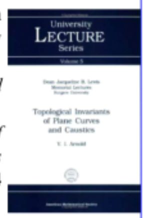
$\twoheadrightarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

"Multi-commutator" (MC) relations



$\longrightarrow$ (a chord-diagram-level drawing)

A Lower Bound on $(\mathcal{K}_0/\mathcal{K}_{n+1})^*$.



References. The root, of course, is [Ar]. Further references on doodles include [Kh, FT, Me, Ta, Va1, Va2]. On Goussarov finite-type: [Go, BN].

[Ar] V.I. Arnold, *Topological Invariants of Plane Curves and Caustics,* American Mathematical Society, 1994.

[BN] D. Bar-Natan, *Bracelets and the Goussarov filtration of the space of knots, Invariants of knots and 3-manifolds (Kyoto 2001),* Geometry and Topology Monographs **4** 1–12, arXiv:math.GT/0111267.

[FT] R. Fenn and P. Taylor, *Introducing Doodles,* in *Topology of Low-Dimensional Manifolds, Proceedings of the Second Sussex Conference, 1977*, Springer 1979.

[Go] M. Goussarov, *Interdependent modifications of links and invariants of finite degree,* Topology **37-3** (1998) 595–602.

[Kh] M. Khovanov, *Doodle Groups,* Trans. Amer. Math. Soc. **349-6** (1997) 2297–2315.

[Me] A.B. Merkov, *Vassiliev Invariants Classify Plane Curves and Doodles,* Sbornik: Mathematics **194-9** (2003) 1301.

[Ta] S. Tabachnikov, *Invariants of Smooth Triple Point Free Plane Curves,* Jour. of Knot Theory and its Ramifications **5-4** (1996) 531–552.

[Va1] V.A. Vassiliev, *On Finite Order Invariants of Triple Point Free Plane Curves,* 1999 preprint, arXiv:1407.7227.

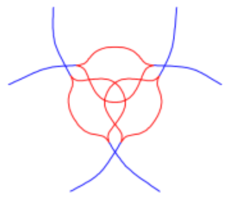
[Va2] V.A. Vassiliev, *Invariants of Ornaments,* Adv. in Soviet Math. **21** (1994) 225–262.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org

Notes / Safekeeping / Recycling.



Notes.

- Where does Arnold's strangeness fit in?
- Perhaps I should put in the Merkov constructions?
- Nearby objects: "virtual doodles", doodles with dots, planar graphs of various kinds, flat braids.

Jonathan's Comment. It seems that the configuration space integrals we defined are more naturally invariants of virtual doodles. Virtual doodles are doodles with some ordinary crossings and some virtual crossings, with the relation that triple points having three virtual crossings are allowed. (Caution: virtual doodles are not Gauss diagrams modulo Reidemeister 2.)

The integrals we defined are invariants for virtual doodles, if we use the rule that the Gauss diagram skeleton is not allowed to use virtual crossings.

What kinds of chords do our integrals detect? They detect "semi-virtuals with outer rings".

(Conjectured) Punchline: Relations on Feynman diagrams correspond with relations on chord diagrams. This is just a matter of carefully checking the analogues of the relations we already knew. What makes this work here and not in the original theory is that we have degree 2 chords, the semi-virtuals.

What would be nice is a clean formulation of finite type for virtual doodles yielding chords which are "semi-virtuals with outer rings".