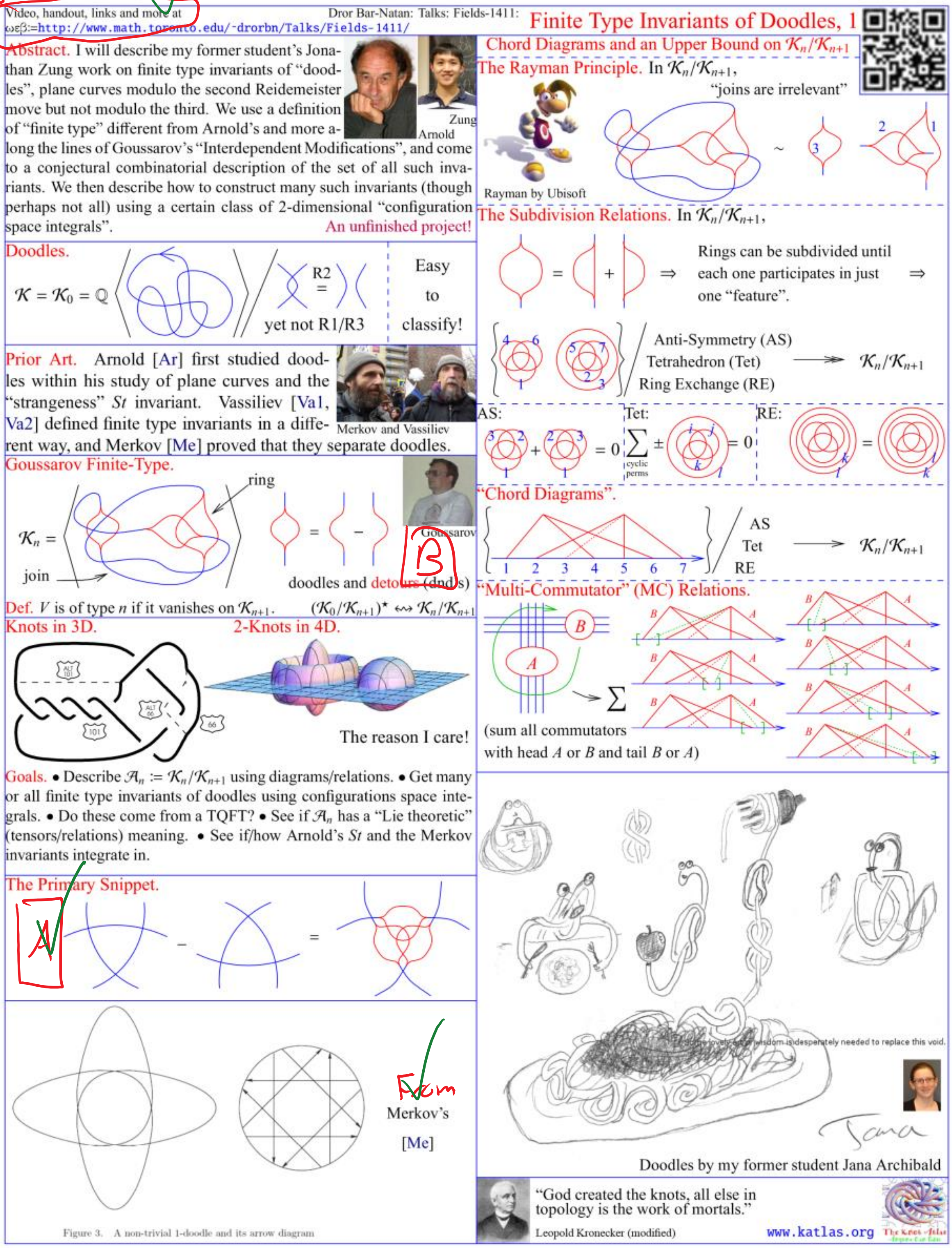
Doodles handout on Nov 17, 2014

November-17-14 9:43 AM

Less info



Video, handout, links and more at ωεβ:=http://www.math.toronto.edu/~drorbn/Talks/Fields-1411/

Dror Bar-Natan: Talks: Fields-1411:

Finite Type Invariants of Doodles, 1

Abstract. I will describe my former student's Jonathan Zung work on finite type invariants of "doodles", plane curves modulo the second Reidemeister move but not modulo the third. We use a definition of "finite type" different from Arnold's and more along the lines of Goussarov's "Interdependent Modifications", and come to a conjectural combinatorial description of the set of all such invariants. We then describe how to construct many such invariants (though perhaps not all) using a certain class of 2-dimensional "configuration space integrals". An unfinished project!

Zung

Arnold

Doodles. $\mathcal{K} = \mathcal{K}_0 = \mathbb{Q}\langle \quad \rangle / \quad \overset{R2}{=}$ yet not R1/R3 | Easy to classify!

Prior Art. Arnold [Ar] first studied doodles within his study of plane curves and the "strangeness" St invariant. Vassiliev [Va1, Va2] defined finite type invariants in a different way, and Merkov [Me] proved that they separate doodles.

Merkov and Vassiliev

Goussarov Finite-Type. $\mathcal{K}_n =$ ring join doodles and detours (dnds) Goussarov

B

Def. V is of type n if it vanishes on $\mathcal{K}_{n+1}$. $(\mathcal{K}_0/\mathcal{K}_{n+1})^\star \leftrightsquigarrow \mathcal{K}_n/\mathcal{K}_{n+1}$

Knots in 3D. **2-Knots in 4D.**

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The reason I care!

Goals. • Describe $\mathcal{A}_n := \mathcal{K}_n/\mathcal{K}_{n+1}$ using diagrams/relations. • Get many or all finite type invariants of doodles using configurations space integrals. • Do these come from a TQFT? • See if $\mathcal{A}_n$ has a "Lie theoretic" (tensors/relations) meaning. • See if/how Arnold's St and the Merkov invariants integrate in.

The Primary Snippet.

A

From Merkov's [Me]

Figure 3. A non-trivial 1-doodle and its arrow diagram

Chord Diagrams and an Upper Bound on $\mathcal{K}_n/\mathcal{K}_{n+1}$

The Rayman Principle. In $\mathcal{K}_n/\mathcal{K}_{n+1}$, "joins are irrelevant"

Rayman by Ubisoft

~ 3 2 1

The Subdivision Relations. In $\mathcal{K}_n/\mathcal{K}_{n+1}$,

= + ⇒ Rings can be subdivided until each one participates in just one "feature". ⇒

Anti-Symmetry (AS)
Tetrahedron (Tet)
Ring Exchange (RE) → $\mathcal{K}_n/\mathcal{K}_{n+1}$

AS: + = 0 | Tet: $\sum_{\text{cyclic perms}} \pm = 0$ | RE: =

"Chord Diagrams". 1 2 3 4 5 6 7 / AS Tet RE → $\mathcal{K}_n/\mathcal{K}_{n+1}$

"Multi-Commutator" (MC) Relations.

B A → $\sum$ (sum all commutators with head A or B and tail B or A)

[illegible] lovely bit of wisdom is desperately needed to replace this void.

Jana

Doodles by my former student Jana Archibald

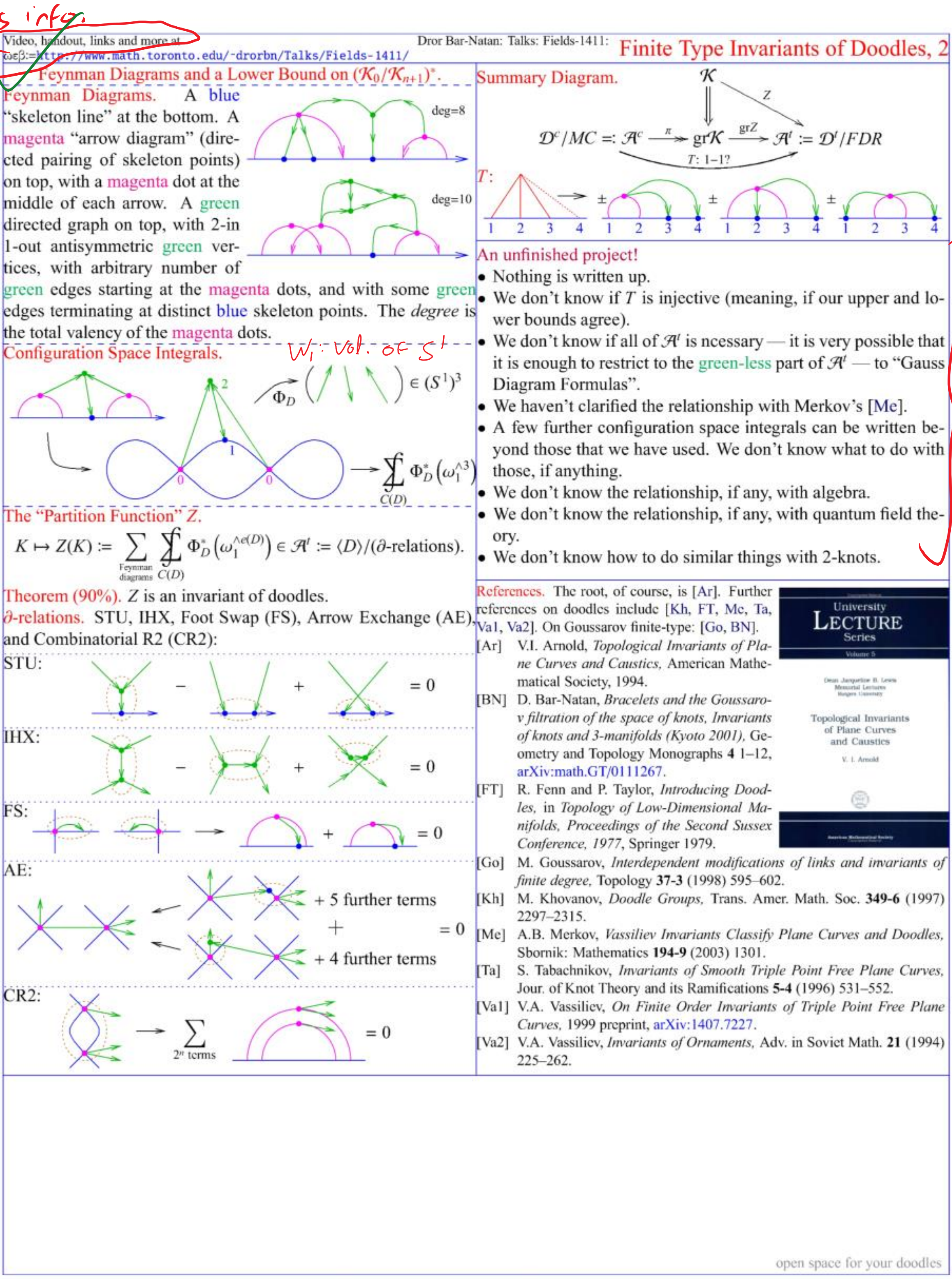
"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

www.katlas.org The Knot Atlas

A: ⇒ Arnold's "strangeness" [Ar] is of type 3.

B: "fixed rotation numbers"

less info



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Finite Type Invariants of Doodles, 2

Feynman Diagrams and a Lower Bound on $(\mathcal{K}_0/\mathcal{K}_{n+1})^*$.

Feynman Diagrams. A blue "skeleton line" at the bottom. A magenta "arrow diagram" (directed pairing of skeleton points) on top, with a magenta dot at the middle of each arrow. A green directed graph on top, with 2-in 1-out antisymmetric green vertices, with arbitrary number of green edges starting at the magenta dots, and with some green edges terminating at distinct blue skeleton points. The *degree* is the total valency of the magenta dots.

Configuration Space Integrals.

W_1: vol. of S^1

$$\xrightarrow{\Phi_D} \left(/ \ \backslash \ \backslash \right) \in (S^1)^3$$

$$\longrightarrow \oint_{C(D)} \Phi_D^*\left(\omega_1^{\wedge 3}\right)$$

The "Partition Function" Z.

$$K \mapsto Z(K) := \sum_{\substack{\text{Feynman}\\ \text{diagrams}}} \oint_{C(D)} \Phi_D^*\left(\omega_1^{\wedge e(D)}\right) \in \mathcal{A}^t := \langle D\rangle/(\partial\text{-relations}).$$

Theorem (90%). Z is an invariant of doodles.

∂-relations. STU, IHX, Foot Swap (FS), Arrow Exchange (AE), and Combinatorial R2 (CR2):

STU: − + = 0

IHX: − + = 0

FS: ⟶ + = 0

AE: + 5 further terms + + 4 further terms = 0

CR2: ⟶ $\sum_{2^n \text{ terms}}$ = 0

Summary Diagram.

$$\begin{tikzcd} & \mathcal{K} \arrow[d, Rightarrow] \arrow[dr, "Z"] & \\ \mathcal{D}^c/MC =: \mathcal{A}^c \arrow[r, two heads, "\pi"] \arrow[rr, bend right, "T:\ 1\text{–}1?"'] & \operatorname{gr}\mathcal{K} \arrow[r, "\operatorname{gr}Z"] & \mathcal{A}^t := \mathcal{D}^t/FDR \end{tikzcd}$$

T: ⟶ ± ± ±

An unfinished project!

- Nothing is written up.
- We don't know if T is injective (meaning, if our upper and lower bounds agree).
- We don't know if all of $\mathcal{A}^t$ is ncessary — it is very possible that it is enough to restrict to the green-less part of $\mathcal{A}^t$ — to "Gauss Diagram Formulas".
- We haven't clarified the relationship with Merkov's [Me].
- A few further configuration space integrals can be written beyond those that we have used. We don't know what to do with those, if anything.
- We don't know the relationship, if any, with algebra.
- We don't know the relationship, if any, with quantum field theory.
- We don't know how to do similar things with 2-knots.

compress?

References. The root, of course, is [Ar]. Further references on doodles include [Kh, FT, Me, Ta, Va1, Va2]. On Goussarov finite-type: [Go, BN].

[Ar] V.I. Arnold, *Topological Invariants of Plane Curves and Caustics,* American Mathematical Society, 1994.

[BN] D. Bar-Natan, *Bracelets and the Goussarov filtration of the space of knots, Invariants of knots and 3-manifolds (Kyoto 2001),* Geometry and Topology Monographs **4** 1–12, arXiv:math.GT/0111267.

[FT] R. Fenn and P. Taylor, *Introducing Doodles,* in *Topology of Low-Dimensional Manifolds, Proceedings of the Second Sussex Conference, 1977*, Springer 1979.

[Go] M. Goussarov, *Interdependent modifications of links and invariants of finite degree,* Topology **37-3** (1998) 595–602.

[Kh] M. Khovanov, *Doodle Groups,* Trans. Amer. Math. Soc. **349-6** (1997) 2297–2315.

[Me] A.B. Merkov, *Vassiliev Invariants Classify Plane Curves and Doodles,* Sbornik: Mathematics **194-9** (2003) 1301.

[Ta] S. Tabachnikov, *Invariants of Smooth Triple Point Free Plane Curves,* Jour. of Knot Theory and its Ramifications **5-4** (1996) 531–552.

[Va1] V.A. Vassiliev, *On Finite Order Invariants of Triple Point Free Plane Curves,* 1999 preprint, arXiv:1407.7227.

[Va2] V.A. Vassiliev, *Invariants of Ornaments,* Adv. in Soviet Math. **21** (1994) 225–262.

open space for your doodles