Becker: Khovanov-Rozansky

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__Def.__ S ring, $w \in S$ "potential", "Linear factorizations" & "Matrix factorization"

__Examples:__ * Suppose $M \in S/w$-Mod

$$\left(0 \rightleftarrows M\right) \in LF(S/w, w)$$

* $x, y \in S$:

$$\{x, y\} := \left(S \underset{\cdot y}{\overset{\cdot x}{\rightleftarrows}} S \right) \in MF(S, xy)$$

Have $\otimes$ products!

$$LF(S,w) \otimes LF(S,w') \subset LF(S, w+w')$$

__Def__ If $w=0$, homology makes sense:

$$H^t\left(M^1 \underset{f}{\overset{g}{\rightleftarrows}} M^0\right) = \begin{matrix} \ker(f)/\operatorname{im}(g) \\ \oplus \\ \ker(g)/\operatorname{im}(f) \end{matrix}$$

If S is graded and w is homogeneous of deg d, put

$$H^t(M^\cdot) := H^0(M^\cdot) \oplus H^1(M^\cdot)\langle \tfrac{d}{2} \rangle$$

K-R by:

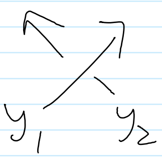
$$\uparrow^{x}_{y} \to \left(\cdots \to 0 \longrightarrow \underline{\hat{\uparrow}} \longrightarrow 0 \to \right) \in \operatorname{Kom}(MF(\mathbb{Q}[x,y], x^{k+1}-y^{k+1}))$$

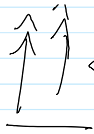
homological deg 0

where

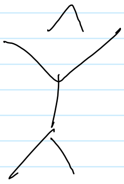
$$\hat{\uparrow} = \left\{ x-y, \frac{x^{k+1}-y^{k+1}}{x-y} \right\}$$

x_1 x_2 ^



$\to \Big(\cdots \to 0 \to$  $\to$  $\langle 1-k \rangle \to 0 \to \cdots$

$$\in \mathrm{Kom}\Big(MF\big(\mathbb{Q}[x_1, x_2, y_1, y_2],\ \underbrace{x_1^{k+1} + x_2^{k+1} - y_1^{k+1} - y_2^{k+1}}_{= W}\big)\Big)$$

where  $:= \begin{matrix} (x_1 + x_2 - y_1 - y_2,\ a) \\ \otimes \\ (x_1 x_2 - y_1 y_2,\ b) \end{matrix}$

where a & b are s.t.

$$(x_1 + x_2 - y_1 - y_2)a + (x_1 x_2 - y_1 y_2)b = W$$

(the specific choice does not matter)

 $\to$

(then tensor all, then take homology)

<u>Thm</u> (Khovanov-Rozansky) This gives an invariant of oriented links categorifying $sl(k)$-quantum invariants.

Matrix Factorization as a derived Category

<u>Analogy</u> $MF(S, W)$ is similar to $K^-(\mathrm{proj}\, R)$

For studying $K^-(\mathrm{proj}(R))$ we have tools:

a.

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