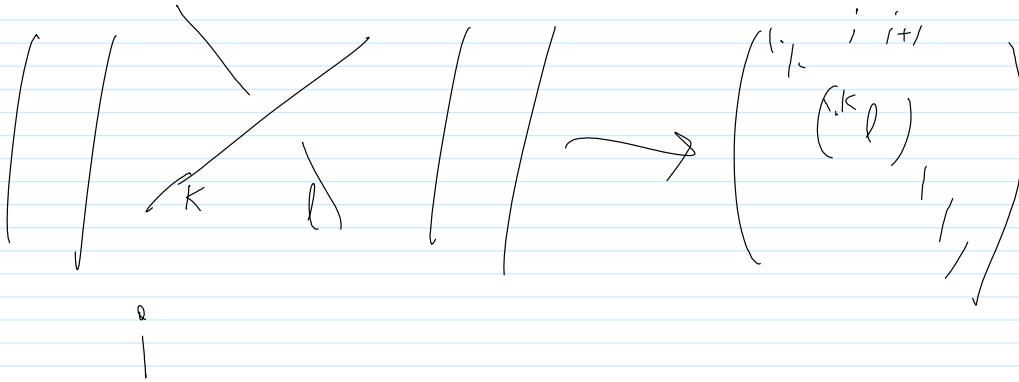



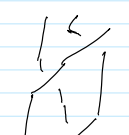
# The most general Gassner invariant

July-14-14 6:14 AM





$$U_1(t_1, t_2) U_2(t_1, t_3) U_1(t_2, t_3)$$

$$=$$


$$U_2(t_2, t_3) U_1(t_1, t_3) U_2(t_1, t_2)$$

where

$$U_1(u, v) = \begin{pmatrix} \alpha(u, v) & \beta(u, v) & 0 \\ \gamma(u, v) & \delta(u, v) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{and} \quad U_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \delta & \delta \end{pmatrix}$$

$$\begin{pmatrix} \alpha_{12} & \beta_{12} & 0 \\ \gamma_{12} & \delta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha_{13} & \beta_{13} \\ 0 & \gamma_{13} & \delta_{13} \end{pmatrix} \begin{pmatrix} \alpha_{23} & \beta_{23} & 0 \\ \gamma_{23} & \delta_{23} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \alpha_{12}\alpha_{23} + \beta_{12}\gamma_{23} & \alpha_{12}\beta_{23} + \beta_{12}\delta_{23} & \beta_{12}\beta_{13} \\ \gamma_{12}\alpha_{23} + \delta_{12}\gamma_{23} & \gamma_{12}\beta_{23} + \delta_{12}\delta_{23} & \delta_{12}\beta_{13} \\ \gamma_{13}\gamma_{23} & \gamma_{13}\delta_{23} & \delta_{13} \end{pmatrix}$$