Cheat Sheet β on July 23, 2014

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Cheat Sheet β

verification at 2014-06/CheatSheetBeta-Verification.nb, 2014-05/GoodFormulas/Demo.nb
http://drorbn.net/AcademicPensieve/2014-07/; initiated 24/3/13; continues 2014-06; modified 13/7/14, 5:51am

σ calculus. $\sigma_1 * \sigma_2 = \sigma_1 \cup \sigma_2$, $tm^{uv}_w = (T_u, T_v \to T_w)$, $hm^{xy}_z : \sigma \mapsto (\sigma \backslash \{x,y\}) \cup (z \to \sigma_x\sigma_y)$, $tha^{ux} = I$, $R^{\pm}_{ux} \mapsto T_u^{\pm 1}$

β-calculus. Constraints. • Sum of column x is $\sigma_x - 1$. • At $T_* = 1$, $\omega = 1$ and $A = 0$.

$$\begin{array}{c|c}\omega_1 & H_1\\ \hline T_1 & A_1\end{array} * \begin{array}{c|c}\omega_2 & H_2\\ \hline T_2 & A_2\end{array} \underset{\beta}{=} \begin{array}{c|cc}\omega_1\omega_2 & H_1 & H_2\\ \hline T_1 & A_1 & 0\\ T_2 & 0 & A_2\end{array} \qquad \begin{array}{c|c}\omega & H\\ \hline u & \alpha\\ v & \beta\\ T & \Xi\end{array} \xrightarrow[\beta]{tm^{uv}_w} \left(\begin{array}{c|c}\omega & H\\ \hline w & \alpha+\beta\\ T & \Xi\end{array}\right)_{T_u,T_v\to T_w} \qquad \begin{array}{c|ccc}\omega & x & y & H\\ \hline T & \alpha & \beta & \Xi\end{array} \xrightarrow[\beta]{hm^{xy}_z} \begin{array}{c|cc}\omega & z & H\\ \hline T & \alpha+\sigma_x\beta & \Xi\end{array}$$

$$\begin{array}{c|cc}\omega & x & H\\ \hline u & \alpha & \theta\\ T & \phi & \Xi\end{array} \xrightarrow[\substack{\beta\\ \nu:=1+\alpha}]{tha^{ux}} \begin{array}{c|cc}\nu\omega & x & H\\ \hline u & \sigma_x\alpha/\nu & \sigma_x\theta/\nu\\ T & \phi/\nu & \Xi-\phi\theta/\nu\end{array} \qquad \rho^{\pm}_{ux} \underset{\beta}{=} \begin{array}{c|c}1 & x\\ \hline u & T_u^{\pm1}-1\end{array}$$

Gassner calculus Γ. Preserves $C_1 :=$ [col sum = 1] ($\Leftrightarrow$OC) and ✓ $C_2 := [\forall a, b,\ (T_a - 1) \mid (A_{ab} - \delta_{ab}\sigma_b)]$

$$\left(\begin{array}{c|cc}\nu\omega & c & S\\ \hline c & \beta+\alpha\delta/\nu & \theta+\alpha\epsilon/\nu\\ S & \psi+\delta\phi/\nu & \Xi+\epsilon\phi/\nu\end{array}\right)_{T_a,T_b\to T_c} \xleftarrow[\nu:=1-\gamma]{m^{ba}_c} \begin{array}{c|ccc}\omega & a & b & S\\ \hline a & \alpha & \beta & \theta\\ b & \gamma & \delta & \epsilon\\ S & \phi & \psi & \Xi\end{array} \xrightarrow[\mu:=1-\beta]{m^{ab}_c} \left(\begin{array}{c|cc}\mu\omega & c & S\\ \hline c & \gamma+\alpha\delta/\mu & \epsilon+\delta\theta/\mu\\ S & \phi+\alpha\psi/\mu & \Xi+\psi\theta/\mu\end{array}\right)_{T_a,T_b\to T_c} \qquad R^{\pm}_{ab} \underset{\gamma}{=} \begin{array}{c|cc}1 & a & b\\ \hline a & 1 & 1-T_a^{\pm1}\\ b & 0 & T_a^{\pm1}\end{array}$$

verify & add the "Fox alternative"

$$\begin{array}{c|cc}\omega & a & S\\ \hline a & \alpha & \theta\\ S & \phi & \Xi\end{array} \xrightarrow[\substack{\mu:=T_a-1\\ \nu:=\alpha-\sigma_a}]{q\Delta^a_{bc}} \left(\begin{array}{c|ccc}\omega & b & c & S\\ \hline b & (\sigma_a-\alpha T_a-\nu T_c)/\mu & (T_b-1)T_c\nu/\mu & (T_b-1)T_c\theta/\mu\\ c & (T_c-1)\nu/\mu & (\alpha-\sigma_aT_a-\nu T_c)/\mu & (T_c-1)\theta/\mu\\ S & \phi & \phi & \Xi\end{array}\right)_{T_a\mapsto T_bT_c}$$

Satisfies: ✓ $R^+_{13}/\!\!/q\Delta^1_{12} = R^+_{23}\#R^+_{13}$. ✓ $R^-_{13}/\!\!/q\Delta^1_{12} = R^-_{13}\#R^-_{23}$. ✓ $q\Delta^a_{a_1a_2}/\!\!/q\Delta^b_{b_1b_2}/\!\!/m^{a_1b_1}_{c_1}/\!\!/m^{a_2b_2}_{c_2} = m^{ab}_c/\!\!/q\Delta^c_{c_1c_2}$.

$$\begin{array}{c|cc}\omega & a & S\\ \hline a & \alpha & \theta\\ S & \phi & \Xi\end{array} \xrightarrow{dS^a} \left(\begin{array}{c|cc}\alpha\omega/\sigma_a & a & S\\ \hline a & 1/\alpha & \theta/\alpha\\ S & -\phi/\alpha & (\alpha\Xi-\phi\theta)/\alpha\end{array}\right)_{T_a\to T_a^{-1}}$$

Satisfies: ✓ $R^{\pm}_{12}/\!\!/dS^{1\text{ or }2} = R^{\mp}_{12}$. ✓ $dm^{ab}_c/\!\!/dS^c = dS^a/\!\!/dS^b/\!\!/dm^{ba}_c$. ✓ $dS^a/\!\!/dS^a = I$. ✓ $q\Delta^a_{bc}/\!\!/dS^b/\!\!/dS^c = dS^a/\!\!/q\Delta^a_{cb}$. ✓ Assuming C_2, $d\eta^a/\!\!/d\epsilon_a = q\Delta^a_{bc}/\!\!/dS^c/\!\!/dm^{bc}_a$ (also 3 variants).

The map (tangle $T \mapsto$ matrix A) is anti-multiplicative. The MVA mod units: $L \mapsto (\omega, A) \mapsto \omega\det'(A-I)/(1-T')$ ✓

add "$\bar{\Gamma}'$" w/ better divisibilities.

Burau. On $b \in uB_n$, $Bu : \sigma_i^{\pm1} \mapsto U_i^{\pm1}$. **Unitarity.** With $U = Bu(b)$, $\boxed{\bar{U}\Omega_nU^T = \Omega_n.}$

add "equivalence of reps" formula

$$\textbf{Thm. } \Gamma(b) = \begin{array}{c|ccc}1 & s_{b(1)} & s_{b(2)} & \cdots\\ \hline s_1 & & & \\ s_2 & & Bu(b)^T & \\ \vdots & & & \end{array}. \qquad U_i = \begin{pmatrix}I_i & & & \\ & 1-t & t & \\ & 1 & 0 & \\ & & & I_{n-i-1}\end{pmatrix},\ U_i^{-1} = \begin{pmatrix}I_i & & & \\ & 0 & 1 & \\ & \bar{t} & 1-\bar{t} & \\ & & & I_{n-i-1}\end{pmatrix},\ \Omega_n = \begin{pmatrix}1 & 0 & \cdots & 0\\ 1-t & 1 & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 1-t & 1-t & \cdots & 1\end{pmatrix}$$

upgrade to Gassner
add "other Gassner" and rel. w/ Fox calculus.

split to Γ_1 (as in Π_1) and Γ_2 (as in Π_2)?

To do. • Full verification program. • Precise relation with Burau/Gassner. • Concordance. • Unitarity. • Planarity. • A depth-mirror property for u-objects. • Mutations? • Link relations? • Behaviour of A/MVA under mirror/strand reversal?

Add R1 behaviour! Add "mirrors" $(\bar{A}^T)^{-1}$.