Oberwolfach Handout on May 6, 2014

✓ Make a handout booklet!

✓ Setup web page.

Abstract. I will describe some very good formulas for a (matrix plus scalar)-valued extension of the Alexander polynnomial to tangles, then say that everything extends to virtual tangles, then roughly to simply knotted balloons and hoops in 4D, then the target space extends to (free Lie algebras plus cyclic words), and the result is a universal finite type of the knotted objects in its domain. Taking a cue from the BF topological quantum field theory, everything should extend (with some modifications) to arbitrary codimension-2 knots in arbitrary dimension and in particular, to arbitrary 2-knots in 4D. But what is really going on is still a mystery.

Theorem 1. $\exists!$ an invariant $\gamma\colon \{\text{pure } S\text{-tangles}\} \to R \times M_{S\times S}(R)$, where $R = R_S = \mathbb{Z}((T_a)_{a\in S})$ is the ring of rational functions in S variables, intertwining

1. $\left(\begin{array}{c|c} \omega_1 & S_1 \\ \hline S_1 & A_1 \end{array}, \begin{array}{c|c} \omega_2 & S_2 \\ \hline S_2 & A_2 \end{array} \right) \xrightarrow{\ \sqcup\ } \begin{array}{c|cc} \omega_1\omega_2 & S_1 & S_2 \\ \hline S_1 & A_1 & 0 \\ S_2 & 0 & A_2 \end{array}$,

2. $\begin{array}{c|ccc} \omega & a & b & S \\ \hline a & \alpha & \beta & \theta \\ b & \gamma & \delta & \epsilon \\ S & \phi & \psi & \Xi \end{array} \xrightarrow[\mu:=1-\beta]{m_c^{ab}} \left(\begin{array}{c|cc} \mu\omega & c & S \\ \hline c & \gamma+\alpha\delta/\mu & \epsilon+\delta\theta/\mu \\ S & \phi+\alpha\psi/\mu & \Xi+\theta\psi/\mu \end{array} \right)_{T_a,T_b\to T_c}$,

and satisfying $\left({}_a\!\nearrow\!_b, {}_b\!\nwarrow\!_a \right) \xrightarrow{\ \gamma\ } \begin{array}{c|cc} 1 & a & b \\ \hline a & 1 & 1-T_a^{\pm 1} \\ b & 0 & T_a^{\pm 1} \end{array}$.

In addition, 1. The Alexander formula
2. The MVA formula
3. A doubling formula.
4. Strand deletion & reversal.

Tangles & why tangles

References.

[BN] D. Bar-Natan, *Balloons and Hoops and their Universal Finite Type Invariant, BF Theory, and an Ultimate Alexander Invariant*, http://www.math.toronto.edu/~drorbn/papers/KBH/, arXiv:1308.1721.

[BND] D. Bar-Natan and Z. Dancso, *Finite Type Invariants of W-Knotted Objects: From Alexander to Kashiwara and Vergne*, http://www.math.toronto.edu/~drorbn/papers/WKO/.

[CR] A. S. Cattaneo and C. A. Rossi, *Wilson Surfaces and Higher Dimensional Knot Invariants*, Commun. in Math. Phys. **256-3** (2005) 513–537, arXiv:math-ph/0210037.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org   The Knot Atlas