Matveev: Dijkgraff-Witten Invariants Over Z/2

May-27-14 5:32 AM

M^n, G finite, B_G, $h \in H^n(B_G, U)$ $U \subset U(1)$

$$Z(M) = \frac{1}{2} \sum_{f \in [M, B_G]} \langle f^*(h), [M] \rangle \in \mathbb{C}$$

Case $n = 3$, $G = \mathbb{Z}/2$, $B_G = \mathbb{RP}^\infty$ $h = \alpha^3$

$$\rightsquigarrow \quad Z(M) = \frac{1}{2} \sum_{f \in [M, B]} (-1)^{\langle f^* \alpha^3, [M] \rangle} \in \mathbb{Z} \cup \{\tfrac{1}{2}\}$$

Quadratic functions: V: v.s over $\mathbb{Z}/2$

$q : V \longrightarrow \mathbb{Z}_2$ is quadratic if

$l_q : V \times V \to \mathbb{Z}$ $l_q(x, y) := q(x+y) - q(x) - q(y)$

is bilinear

Arf(q)

$$\begin{array}{c} \overbrace{a_1 \quad b_1 \quad a_2 \quad b_2}^{k} \; \overbrace{c_1 \quad c_2}^{m} \\ \begin{bmatrix} 0 & 1 & & & & & \\ 1 & 0 & & & & & \\ & & 0 & 1 & & & \\ & & 1 & 0 & & & \\ & & & & 0 & & \\ & & & & & 0 & \\ & & & & & & \end{bmatrix} \end{array}$$

any l_q can be brought to this form.

$$\text{Arf}(q) = \sum_{i=1}^{k} q(a_i) q(b_i)$$

<u>Claim</u> If $q|_{\langle c_i \rangle} = 0$ then Arf is invariant.

$M^3 \rightsquigarrow Q_M : H^1(M; \mathbb{Z}/2) \longrightarrow \mathbb{Z}/2$

q_M.

$Q_M(x) = \langle x^3, [M] \rangle$ is quadratic!

<u>Thm</u> M^3, $A \subset H^1(M;\mathbb{Z}/2)$ the annihilator of l_M

1. If $\exists x \in A$ s.t. $x^3 \neq 0 \Rightarrow Z(M) = 0$
2. If $\forall x \in A$ $x^3 = 0$, then

$$Z(M) = 2^{k+m-1}(-1)^{Arf(Q_M)}$$

D-W

Computation using Gaussian Sums

$$\sigma(q) := \sum_{x \in V} (-1)^{q(x)}$$

1. Multiplicativity $q_i : V_i \to \mathbb{Z}/2$ $i = 1,2$

   $q : V_1 \oplus V_2 \to \mathbb{Z}_2$ the sum, then

$$\sigma(q) = \sigma(q_1) \cdot \sigma(q_2)$$

2. $$\sigma(q) = 2^{k+m}(-1)^{Arf(q)} \prod_{j=1}^{m} (1 - q(c_j))$$

pf: enough to calculate on the blocks

$$\begin{array}{c|cc} & a & b \\ \hline a & 0 & 1 \\ b & 1 & 0 \end{array} \quad \& \quad \begin{array}{c|c} & c \\ \hline c & 0 \end{array}$$

Finally, $Z(M) = \sigma(Q_M)$ .....

$$Z(L_{(p,q)}) = \begin{cases} 0 & p \equiv 2 \mod 4 \\ 1/2 & p \text{ is odd} \\ 1 & p \text{ is divisible by } 4. \end{cases}$$