Pensieve Header: An attempt on the ribbon property using Γ-calculus.

```
dir = SetDirectory["C:/drorbn/AcademicPensieve/2014-05/"];
<< KnotTheory`
<< MetaCalculi/MetaCalculi-Program.m
```

```
Loading KnotTheory` version of April 3, 2014, 16:23:56.0784.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
n = 4; γ0 = Γ[ω, Sum[h_a σ_a, {a, 0, n}], Sum[Sum[t_a h_b α_{10 a+b}, {b, 1, n}], {a, 1, n}]]
```

$$\begin{pmatrix} \omega & S_1 & S_2 & S_3 & S_4 \\ S_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\ S_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} \\ S_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} \\ S_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 \end{pmatrix}$$

```
Ov = Xp[o1, 1] Xp[o2, 2] Xp[o3, 3] Xp[o4, 4] // Γ // dm[o1, o2, o] // dm[o, o3, o] //
  dm[o, o4, o]
```

$$\begin{pmatrix} 1 & S_1 & S_2 & S_3 & S_4 & S_o \\ S_1 & T_o & 0 & 0 & 0 & 0 \\ S_2 & 0 & T_o & 0 & 0 & 0 \\ S_3 & 0 & 0 & T_o & 0 & 0 \\ S_4 & 0 & 0 & 0 & T_o & 0 \\ S_o & 1-T_o & 1-T_o & 1-T_o & 1-T_o & 1 \\ \Sigma & T_o & T_o & T_o & T_o & 1 \end{pmatrix}$$

```
{t1 = Ov ** (γ0 * Γ[ϵ[o]]), t2 = (γ0 * Γ[ϵ[o]]) ** Ov, ocond = Simplify[t1 == t2]}
```

$$\left\{ \begin{pmatrix} \omega & S_1 & S_2 & S_3 & S_4 & S_o \\ S_1 & T_o\,\alpha_{11} & T_o\,\alpha_{12} & T_o\,\alpha_{13} & T_o\,\alpha_{14} & 0 \\ S_2 & T_o\,\alpha_{21} & T_o\,\alpha_{22} & T_o\,\alpha_{23} & T_o\,\alpha_{24} & 0 \\ S_3 & T_o\,\alpha_{31} & T_o\,\alpha_{32} & T_o\,\alpha_{33} & T_o\,\alpha_{34} & 0 \\ S_4 & T_o\,\alpha_{41} & T_o\,\alpha_{42} & T_o\,\alpha_{43} & T_o\,\alpha_{44} & 0 \\ S_o & 1-T_o & 1-T_o & 1-T_o & 1-T_o & 1 \\ \Sigma & T_o\,\sigma_1 & T_o\,\sigma_2 & T_o\,\sigma_3 & T_o\,\sigma_4 & 1 \end{pmatrix}, \begin{pmatrix} \omega & S_1 & S_2 \\ S_1 & T_o\,\alpha_{11} & T_o\,\alpha_{12} \\ S_2 & T_o\,\alpha_{21} & T_o\,\alpha_{22} \\ S_3 & T_o\,\alpha_{31} & T_o\,\alpha_{32} \\ S_4 & T_o\,\alpha_{41} & T_o\,\alpha_{42} \\ S_o & -(-1+T_o)\,(\alpha_{11}+\alpha_{21}+\alpha_{31}+\alpha_{41}) & -(-1+T_o)\,(\alpha_{12}+\alpha_{22} \\ \Sigma & T_o\,\sigma_1 & T_o\,\sigma_2 \end{pmatrix}, \right.$$

$$(-1+T_o)\,(-1+\alpha_{11}+\alpha_{21}+\alpha_{31}+\alpha_{41}) == 0 \;\&\&\; (-1+T_o)\,(-1+\alpha_{12}+\alpha_{22}+\alpha_{32}+\alpha_{42}) == 0 \;\&\&$$
$$\left. (-1+T_o)\,(-1+\alpha_{13}+\alpha_{23}+\alpha_{33}+\alpha_{43}) == 0 \;\&\&\; (-1+T_o)\,(-1+\alpha_{14}+\alpha_{24}+\alpha_{34}+\alpha_{44}) == 0 \right\}$$

```
ocond = Simplify[ocond /. T_o → 0]
```

$$\alpha_{11}+\alpha_{21}+\alpha_{31}+\alpha_{41} == 1 \;\&\&\; \alpha_{12}+\alpha_{22}+\alpha_{32}+\alpha_{42} == 1 \;\&\&\; \alpha_{13}+\alpha_{23}+\alpha_{33}+\alpha_{43} == 1 \;\&\&\; \alpha_{14}+\alpha_{24}+\alpha_{34}+\alpha_{44} == 1$$

```
U = Xm[1, u1] Xm[2, u2] Xm[3, u3] Xm[4, u4] // Γ // dm[u1, u2, u] // dm[u, u3, u] //
  dm[u, u4, u]
```

	1	s_1	s_2	s_3	s_4	s_u
s_1	1	0	0	0	$\frac{-1+T_1}{T_1}$
s_2	0	1	0	0	$\frac{-1+T_2}{T_1 T_2}$
s_3	0	0	1	0	$\frac{-1+T_3}{T_1 T_2 T_3}$
s_4	0	0	0	1	$\frac{-1+T_4}{T_1 T_2 T_3 T_4}$
s_u	0	0	0	0	$\frac{1}{T_1 T_2 T_3 T_4}$
Σ	1	1	1	1	$\frac{1}{T_1 T_2 T_3 T_4}$

```
{t1 = U ** (γ0 * Γ[ϵ[u]]), t2 = (γ0 * Γ[ϵ[u]]) ** U, ucond = Simplify[t1 == t2]}
```

$$\left\{\begin{array}{c|cccc|c} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ \hline s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-T_2 T_3 T_4 \alpha_{11}+T_1 T_2 T_3 T_4 \alpha_{11}-T_3 T_4 \alpha_{12}+T_2 T_3 T_4 \alpha_{12}-T_4 \alpha_{13}+T_3 T_4 \alpha_{13}-\alpha_{14}+T_4 \alpha_{14}}{T_1 T_2 T_3 T_4} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-T_2 T_3 T_4 \alpha_{21}+T_1 T_2 T_3 T_4 \alpha_{21}-T_3 T_4 \alpha_{22}+T_2 T_3 T_4 \alpha_{22}-T_4 \alpha_{23}+T_3 T_4 \alpha_{23}-\alpha_{24}+T_4 \alpha_{24}}{T_1 T_2 T_3 T_4} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-T_2 T_3 T_4 \alpha_{31}+T_1 T_2 T_3 T_4 \alpha_{31}-T_3 T_4 \alpha_{32}+T_2 T_3 T_4 \alpha_{32}-T_4 \alpha_{33}+T_3 T_4 \alpha_{33}-\alpha_{34}+T_4 \alpha_{34}}{T_1 T_2 T_3 T_4} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-T_2 T_3 T_4 \alpha_{41}+T_1 T_2 T_3 T_4 \alpha_{41}-T_3 T_4 \alpha_{42}+T_2 T_3 T_4 \alpha_{42}-T_4 \alpha_{43}+T_3 T_4 \alpha_{43}-\alpha_{44}+T_4 \alpha_{44}}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array}\right\},$$

$$\left\{\begin{array}{c|cccc|c} \omega & s_1 & s_2 & s_3 & s_4 & s_u \\ \hline s_1 & \alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} & \frac{-1+T_1}{T_1} \\ s_2 & \alpha_{21} & \alpha_{22} & \alpha_{23} & \alpha_{24} & \frac{-1+T_2}{T_1 T_2} \\ s_3 & \alpha_{31} & \alpha_{32} & \alpha_{33} & \alpha_{34} & \frac{-1+T_3}{T_1 T_2 T_3} \\ s_4 & \alpha_{41} & \alpha_{42} & \alpha_{43} & \alpha_{44} & \frac{-1+T_4}{T_1 T_2 T_3 T_4} \\ s_u & 0 & 0 & 0 & 0 & \frac{1}{T_1 T_2 T_3 T_4} \\ \Sigma & \sigma_1 & \sigma_2 & \sigma_3 & \sigma_4 & \frac{1}{T_1 T_2 T_3 T_4} \end{array}\right\},$$

$$\begin{aligned} &\frac{1}{T_1 T_2 T_3 T_4}\left(T_2 T_3 T_4 \left(1+\left(-1+T_1\right)\alpha_{11}+\alpha_{12}\right)-T_4 \alpha_{13}+T_3 T_4 \left(-\alpha_{12}+\alpha_{13}\right)-\alpha_{14}+T_4 \alpha_{14}\right)==1 \&\& \\ &\frac{1}{T_1 T_2 T_3 T_4}\left(T_3 T_4 \left(1-\alpha_{22}+T_2 \left(-1+\left(-1+T_1\right)\alpha_{21}+\alpha_{22}\right)+\alpha_{23}\right)-\alpha_{24}+T_4 \left(-\alpha_{23}+\alpha_{24}\right)\right)==0 \&\& \\ &\frac{1}{T_1 T_2 T_3 T_4}\left(-\alpha_{34}+T_4 \left(1-\alpha_{33}+T_3 \left(-1-\alpha_{32}+T_2 \left(\left(-1+T_1\right)\alpha_{31}+\alpha_{32}\right)+\alpha_{33}\right)+\alpha_{34}\right)\right)==0 \&\& \\ &\frac{1}{T_1 T_2 T_3 T_4}\left(1-\alpha_{44}+T_4 \left(-1+T_2 T_3 \left(\left(-1+T_1\right)\alpha_{41}+\alpha_{42}\right)-\alpha_{43}+T_3 \left(-\alpha_{42}+\alpha_{43}\right)+\alpha_{44}\right)\right)==0\Big\} \end{aligned}$$

```
γ1 = (γ0 // dS[2] // dS[4])
```

$$\begin{pmatrix} -\frac{\omega\,(\alpha_{24}\,\alpha_{42}-\alpha_{22}\,\alpha_{44})}{\sigma_2\,\sigma_4} & S_1 & S_2 & \\ S_1 & \frac{\alpha_{14}\,\alpha_{22}\,\alpha_{41}-\alpha_{12}\,\alpha_{24}\,\alpha_{41}-\alpha_{14}\,\alpha_{21}\,\alpha_{42}+\alpha_{11}\,\alpha_{24}\,\alpha_{42}+\alpha_{12}\,\alpha_{21}\,\alpha_{44}-\alpha_{11}\,\alpha_{22}\,\alpha_{44}}{\alpha_{24}\,\alpha_{42}-\alpha_{22}\,\alpha_{44}} & -\frac{\alpha_{14}\,\alpha_{42}-\alpha_{12}\,\alpha_{44}}{\alpha_{24}\,\alpha_{42}-\alpha_{22}\,\alpha_{44}} & \frac{-\alpha_{14}\,\alpha_{23}\,\alpha_{42}+\alpha_{13}\,\alpha_{24}\,\alpha_{42}+\alpha_1}{} \\ S_2 & \frac{\alpha_{24}\,\alpha_{41}-\alpha_{21}\,\alpha_{44}}{\alpha_{24}\,\alpha_{42}-\alpha_{22}\,\alpha_{44}} & \frac{\alpha_{44}}{-\alpha_{24}\,\alpha_{42}+\alpha_{22}\,\alpha_{44}} & \\ S_3 & \frac{-\alpha_{24}\,\alpha_{32}\,\alpha_{41}+\alpha_{22}\,\alpha_{34}\,\alpha_{41}+\alpha_{24}\,\alpha_{31}\,\alpha_{42}-\alpha_{21}\,\alpha_{34}\,\alpha_{42}-\alpha_{22}\,\alpha_{31}\,\alpha_{44}+\alpha_{21}\,\alpha_{32}\,\alpha_{44}}{\alpha_{24}\,\alpha_{42}-\alpha_{22}\,\alpha_{44}} & -\frac{\alpha_{34}\,\alpha_{42}-\alpha_{32}\,\alpha_{44}}{\alpha_{24}\,\alpha_{42}-\alpha_{22}\,\alpha_{44}} & \frac{\alpha_{24}\,\alpha_{33}\,\alpha_{42}-\alpha_{23}\,\alpha_{34}\,\alpha_{42}-\alpha_{24}}{} \\ S_4 & \frac{\alpha_{22}\,\alpha_{41}-\alpha_{21}\,\alpha_{42}}{-\alpha_{24}\,\alpha_{42}+\alpha_{22}\,\alpha_{44}} & \frac{\alpha_{42}}{\alpha_{24}\,\alpha_{42}-\alpha_{22}\,\alpha_{44}} & \\ \Sigma & \sigma_1 & \frac{1}{\sigma_2} & \end{pmatrix}$$

```
cert = γ1 // dm[1, 2, 1] // dm[3, 4, 2]
```

$$\begin{pmatrix} -\frac{\omega\,(\alpha_{14}\,\alpha_{32}+\alpha_{24}\,\alpha_{32}-\alpha_{12}\,\alpha_{34}-\alpha_{22}\,\alpha_{34}+\alpha_{14}\,\alpha_{42}+\alpha_{24}\,\alpha_{42}-\alpha_{12}\,\alpha_{44}-\alpha_{22}\,\alpha_{44})}{\sigma_2\,\sigma_4} & S_1 & \\ S_1 & \frac{\alpha_{14}\,\alpha_{31}+\alpha_{24}\,\alpha_{31}-\alpha_{11}\,\alpha_{34}-\alpha_{21}\,\alpha_{34}+\alpha_{14}\,\alpha_{41}+\alpha_{24}\,\alpha_{41}-\alpha_{11}\,\alpha_{44}-\alpha_{21}\,\alpha_{44}}{\alpha_{14}\,\alpha_{32}+\alpha_{24}\,\alpha_{32}-\alpha_{12}\,\alpha_{34}-\alpha_{22}\,\alpha_{34}+\alpha_{14}\,\alpha_{42}+\alpha_{24}\,\alpha_{42}-\alpha_{12}\,\alpha_{44}-\alpha_{22}\,\alpha_{44}} & \frac{\alpha_{14}\,\alpha}{\alpha_{14}\,\alpha} \\ S_2 & \frac{\alpha_{12}\,\alpha_{31}+\alpha_{22}\,\alpha_{31}-\alpha_{11}\,\alpha_{32}-\alpha_{21}\,\alpha_{32}+\alpha_{12}\,\alpha_{41}+\alpha_{22}\,\alpha_{41}-\alpha_{11}\,\alpha_{42}-\alpha_{21}\,\alpha_{42}}{-\alpha_{14}\,\alpha_{32}-\alpha_{24}\,\alpha_{32}+\alpha_{12}\,\alpha_{34}+\alpha_{22}\,\alpha_{34}-\alpha_{14}\,\alpha_{42}-\alpha_{24}\,\alpha_{42}+\alpha_{12}\,\alpha_{44}+\alpha_{22}\,\alpha_{44}} & \frac{\alpha_{13}\,\alpha}{\alpha_{14}\,\alpha} \\ \Sigma & \frac{\sigma_1}{\sigma_2} & \end{pmatrix}$$

```
eqns = (ϵ[1] ϵ[2] // Γ) == (cert /. σ_ → 1) // Simplify
```

$$\begin{aligned} &-\omega\,(\alpha_{14}\,(\alpha_{32}+\alpha_{42})+\alpha_{24}\,(\alpha_{32}+\alpha_{42})-(\alpha_{12}+\alpha_{22})\,(\alpha_{34}+\alpha_{44}))==1\;\&\&\\ &\frac{\alpha_{14}\,(\alpha_{31}+\alpha_{41})+\alpha_{24}\,(\alpha_{31}+\alpha_{41})-(\alpha_{11}+\alpha_{21})\,(\alpha_{34}+\alpha_{44})}{\alpha_{14}\,(\alpha_{32}+\alpha_{42})+\alpha_{24}\,(\alpha_{32}+\alpha_{42})-(\alpha_{12}+\alpha_{22})\,(\alpha_{34}+\alpha_{44})}==1\;\&\&\\ &\frac{\alpha_{14}\,(\alpha_{33}+\alpha_{43})+\alpha_{24}\,(\alpha_{33}+\alpha_{43})-(\alpha_{13}+\alpha_{23})\,(\alpha_{34}+\alpha_{44})}{\alpha_{14}\,(\alpha_{32}+\alpha_{42})+\alpha_{24}\,(\alpha_{32}+\alpha_{42})-(\alpha_{12}+\alpha_{22})\,(\alpha_{34}+\alpha_{44})}==0\;\&\&\\ &\frac{\alpha_{12}\,(\alpha_{31}+\alpha_{41})+\alpha_{22}\,(\alpha_{31}+\alpha_{41})-(\alpha_{11}+\alpha_{21})\,(\alpha_{32}+\alpha_{42})}{-\alpha_{14}\,(\alpha_{32}+\alpha_{42})-\alpha_{24}\,(\alpha_{32}+\alpha_{42})+(\alpha_{12}+\alpha_{22})\,(\alpha_{34}+\alpha_{44})}==0\;\&\&\\ &\frac{\alpha_{13}\,(\alpha_{32}+\alpha_{42})+\alpha_{23}\,(\alpha_{32}+\alpha_{42})-(\alpha_{12}+\alpha_{22})\,(\alpha_{33}+\alpha_{43})}{\alpha_{14}\,(\alpha_{32}+\alpha_{42})+\alpha_{24}\,(\alpha_{32}+\alpha_{42})-(\alpha_{12}+\alpha_{22})\,(\alpha_{34}+\alpha_{44})}==1 \end{aligned}$$

```
alex = (γ1 // dm[4, 3, 3] // dm[3, 2, 2] // dm[2, 1, 1])[[1]]
```

$$\begin{aligned} &-\frac{1}{\sigma_2\,\sigma_4}\omega\,(\alpha_{23}\,\alpha_{41}-\alpha_{24}\,\alpha_{41}+\alpha_{33}\,\alpha_{41}-\alpha_{34}\,\alpha_{41}-\alpha_{23}\,\alpha_{42}+\alpha_{24}\,\alpha_{42}-\alpha_{33}\,\alpha_{42}+\\ &\qquad\alpha_{34}\,\alpha_{42}-\alpha_{21}\,\alpha_{43}+\alpha_{22}\,\alpha_{43}-\alpha_{31}\,\alpha_{43}+\alpha_{32}\,\alpha_{43}+\alpha_{21}\,\alpha_{44}-\alpha_{22}\,\alpha_{44}+\alpha_{31}\,\alpha_{44}-\alpha_{32}\,\alpha_{44}) \end{aligned}$$

```
FullSimplify[alex, eqns && ucond && ocond]
```

$$\frac{\omega\,(-(\alpha_{23}-\alpha_{24})\,(\alpha_{41}-\alpha_{42})+(\alpha_{21}-\alpha_{22})\,(\alpha_{43}-\alpha_{44}))}{\sigma_2\,\sigma_4}$$

```
FullSimplify[cert[[1]]]
```

$$\frac{\omega\,(-(\alpha_{14}+\alpha_{24})\,(\alpha_{32}+\alpha_{42})+(\alpha_{12}+\alpha_{22})\,(\alpha_{34}+\alpha_{44}))}{\sigma_2\,\sigma_4}$$

```
M = {{-T2/(-T1-T2+T1 T2), (-1+T1)/(-T1-T2+T1 T2), 0},
     {-(T1 (-1+T2) T3)/((-T1-T2+T1 T2) (-T2-T3+T2 T3)), -((1-2 T1-T2+T1 T2) T3)/((-T1-T2+T1 T2) (-T2-T3+T2 T3)), (-1+T2)/(-T2-T3+T2 T3)},
     {(T1 (-1+T2) T2 (-1+T3))/((-T1-T2+T1 T2) (-T2-T3+T2 T3)), (T2 (1-2 T1-T2+T1 T2) (-1+T3))/((-T1-T2+T1 T2) (-T2-T3+T2 T3)), (1-2 T2-T3+T2 T3)/(-T2-T3+T2 T3)}};
```

```
Simplify[M /. T_a_ :> 1 / T_a] // MatrixForm
```

$$\begin{pmatrix} \frac{T_1}{-1+T_1+T_2} & \frac{(-1+T_1)\,T_2}{-1+T_1+T_2} & 0 \\ \frac{(-1+T_2)\,T_2}{(-1+T_1+T_2)\,(-1+T_2+T_3)} & -\frac{(1+T_1\,(-1+T_2)-2\,T_2)\,T_2}{(-1+T_1+T_2)\,(-1+T_2+T_3)} & \frac{(-1+T_2)\,T_3}{-1+T_2+T_3} \\ \frac{(-1+T_2)\,(-1+T_3)}{(-1+T_1+T_2)\,(-1+T_2+T_3)} & -\frac{(1+T_1\,(-1+T_2)-2\,T_2)\,(-1+T_3)}{(-1+T_1+T_2)\,(-1+T_2+T_3)} & \frac{-1-T_2\,(-1+T_3)+2\,T_3}{-1+T_2+T_3} \end{pmatrix}$$

```
Simplify[(M /. T_a_ :> 1 / T_a).Transpose[M]] // Simplify // MatrixForm
```

$$\begin{pmatrix} \frac{\left(1-3\,T_1+T_1^2\right)\,T_2}{(T_1\,(-1+T_2)-T_2)\,(-1+T_1+T_2)} & -\frac{\left(T_1\,(3-2\,T_2)\,T_2+\right.}{(T_1\,(-1+T_2)-T_2)} \\ -\frac{T_2\,\left(-1+T_1\,(2-3\,T_2)+T_1^2\,(-1+T_2)+T_2+T_2^2\right)}{(T_1\,(-1+T_2)-T_2)\,(-1+T_1+T_2)\,(-1+T_2+T_3)} & \frac{\left(-T_2\,\left(-2+6\,T_2-5\,T_2^2+T_2^3\right)+T_1^2\,\left(-1+5\right.\right.}{(T_1\,(-1+T_2)-T_2)\,(-1+T} \\ -\frac{\left(-1+T_1\,(2-3\,T_2)+T_1^2\,(-1+T_2)+T_2+T_2^2\right)\,(-1+T_3)}{(T_1\,(-1+T_2)-T_2)\,(-1+T_1+T_2)\,(-1+T_2+T_3)} & \frac{-T_1^2\,(-1+T_2)\,\left(-1+T_2^2\,(-1+T_3)+2\,T_3^2-T_2\,\left(-2+2\,T_3+T_3^2\right)\right)+(-1+T_2)\,\left(T_2^2\,(2-3\,T_3)+T_2^3\,(-1+T_3)-(\right.}{(T_1\,(-1+T_2)-T_2)\,(-1+T} \end{pmatrix}$$

```
Simplify[M /. T_a_ :> 1 / T_a] / M /. T_ -> T // Simplify // MatrixForm
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

Infinity::indet : Indeterminate expression 0 ComplexInfinity encountered. ≫

$$\begin{pmatrix} -\frac{(-2+T)\,T}{-1+2\,T} & \frac{(-2+T)\,T^2}{-1+2\,T} & \text{Indeterminate} \\ -\frac{(-2+T)^2\,T}{(1-2\,T)^2} & \frac{(-2+T)^2\,T^2}{(1-2\,T)^2} & \frac{(-2+T)\,T^2}{-1+2\,T} \\ \frac{(-2+T)^2}{(1-2\,T)^2} & -\frac{(-2+T)^2\,T}{(1-2\,T)^2} & -\frac{(-2+T)\,T}{-1+2\,T} \end{pmatrix}$$