Abstract: I will describe a semi-exact reduction of perturbation BF-theory (Cattaneo-Morselli [C-M]) to computable combinatorics, in the case of ribbon 2-links. Also, I will explain how and why my approach may or may not work in the non-ribbon case. Such a result is, and at least partially already known (Khovanov [K]). Yet in the ribbon case, the resulting invariant is a universal finite-type invariant, a gadget that significantly generalizes and clarifies the Alexander polynomial and that is closely related to the Khovanov-Vassiliev problem. I cannot rule out the possibility that the corresponding gadget in the non-ribbon case will be of interest.

BF Following (C-M): $A \in D(M = R^n, \beta), B \in D(S, g')$.

Such that $A \in D(M = R^n, \beta), B \in D(S, g')$, set

$D(A, B) := \sum_{(A', B')} [A'] [B']$

With $f: (S = R^d) \to M, \xi \in D(M, \gamma), \beta \in D(S, g'), \gamma$, set

$D(A, B, f) := \sum_{(A', B')} [A'] [B'] [\xi] [\gamma]$
Theorem 2. Using Gauss diagrams to represent knots and 5-component pure tangles, the above formulas define an invariant in $C_w(\mathbb{Z}) \to C_w(\mathbb{Z})$, "cyclic w-balls in $\mathbb{Z}$".

- Agrees with BN-Dancso (BND) and with [BN2].
- In practice: compatible! Variables on braids. Extends to $w = \infty$ (contains Alexander). The "missing factor" is Levine's扎根is (Lek) theorem of the rest of $[L]$ also fits, hence contains the MVA.
- Related to $\widetilde{\varepsilon}$-extension of Farber's $\varepsilon$-fact.
- Should be summed and categorified.

Sketch of Proof. In 4D an algebraic gauge, only "drop down" red propagators, hence in the ribbon case, no $M$-invariant vertices. $5$-integers are a $\varepsilon$ "ground pieces" run on nested curves as below, and $\varepsilon$-invariants arise when several propagators compete for the same double curve. And then the combinatorics is obvious.

**Notes.** In 3D, a generic immersion of $S^1$ is an embedding, a knot. In 4D, a generic immersion of a surface has finitely-many double points (a pair). Perhaps we should be studying these?

**Finite-type: What are finite-type invariants for 2-knots? What would be "diagram diagrams"?**

**Bubble-wrap: What about the $\varepsilon$-invariant (the "true" triple-linking number)?**

**References**

- **Conclusion.** A virtual knot is a group only 2-knot.