Stefan Sakalos: On Quantization of Quasi-Lie Bialgebras

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Bialgebra: \((B, \Delta, E)\) Example: \(U(g)\)

Monoidal Categories:

- Functor \(\otimes: B \times B \to B\), unit 1, associativity
- Pentagon

\(B\text{-Mod}\) is a monoidal category.

Quasi-Bi-Algebra: \((B, \Delta, E, \Phi \in B^{\otimes 3})\)

What should \(\Phi\) satisfy to get a monoidal category?

Ans. The usual suspects.

A deformation of an algebra \(A\): a product on \(B := A \oplus \mathbb{H}\) s.t. \(B/\mathbb{H} \cong A\).

\[
\mathcal{E}[f, g] := \frac{1}{\hbar} [F_i, g] \mod B
\]

is a Poisson structure.

Given a Poisson structure, does it always have a quantization? \\

A deformation of the Bialgebra \(U(g)\) is a "QUE".

A deformation of \(U(g)\) as a quasi-Hopf bialgebra is a "quasi-QUE algebra".

\[
\delta(x) := \frac{1}{\hbar} \delta(x) \mod \mathbb{H}
\]

--- get Lie bialgebras.

A quantization of a Lie-bialgebra \(\ldots\) exists by Etingof- Knizhnan.

Classical limit of a quasi-QUE:

\[
\gamma \sim \frac{1}{\hbar} \text{Alt } \Phi \mod \hbar
\]

\(\gamma\) satisfies some relations \(\ldots\)
Set $P = g \odot g^*$, has metric $<\cdot>$. A quasi Lie bialgebra structure on $g$ is the same as a Lie bracket on $P$ s.t.

1. $g$ is a Lie subalgebra.

2. $<\cdot>$ is an invariant scalar product.

"The Drinfeld double"

Theorem (Enriquez - Habout). Quantizations exist.

A Drinfeld associator for a Lie algebra $P$ w/ metric $\cdots$.

Another method for quantization of quasi-Lie bialgebras...

\cdots we construct quasi-bi-algebras as endomorphisms of "forgetful functors".
