

Dror Bar-Natan: Talks: Geneva-131024 $\omega := \text{http://www.math.toronto.edu/~drorbn/Talks/Geneva-131024}$

Finite Type Invariants of Ribbon Knotted Balloons and Hoops

Abstract. On my September 17 Geneva talk (ω/sep) I described a certain trees-and-wheels-valued invariant ζ of ribbon knotted loops and 2-spheres in 4-space, and my October 8 Geneva talk (ω/oct) describes its reduction to the Alexander polynomial. Today I will explain how that same invariant arises completely naturally within the theory of finite type invariants of ribbon knotted loops and 2-spheres in 4-space.

$\mathcal{K}^{bh} = \mathbb{Q} \langle \text{pictures with } \geq n \text{ semi-virts} \rangle \subset \mathcal{K}^{bh}$

Let $\mathcal{I}^n := \langle \text{pictures with } \geq n \text{ semi-virts} \rangle \subset \mathcal{K}^{bh}$. We seek an "expansion"

$$Z: \mathcal{K}^{bh} \rightarrow \text{gr } \mathcal{K}^{bh} = \widehat{\bigoplus} \mathcal{I}^n / \mathcal{I}^{n+1} =: \mathcal{A}^{bh}$$

satisfying "property U": if $\gamma \in \mathcal{I}^n$, then $Z(\gamma) = (0, \dots, 0, \gamma / \mathcal{I}^{n+1}, *, *, \dots)$.

Why? • Just because, and this is vastly more general.
 • $(\mathcal{K}^{bh} / \mathcal{I}^{n+1})^*$ is "finite-type/polynomial invariants".
 • The Taylor example: Take $\mathcal{K} = C^\infty(\mathbb{R}^n)$, $\mathcal{I} = \{f \in \mathcal{K} : f(0) = 0\}$. Then $\mathcal{I}^n = \{f : f \text{ vanishes like } |x|^n\}$ so $\mathcal{I}^n / \mathcal{I}^{n+1}$ is homogeneous polynomials of degree n and Z is a "Taylor expansion"! (So Taylor expansions are vastly more general than you'd think).

Plan. We'll construct a graded \mathcal{A}^{bh} , a surjective graded $\pi: \mathcal{A}^{bh} \rightarrow \mathcal{A}^{bh}$, and a filtered $\tilde{Z}: \mathcal{K}^{bh} \rightarrow \mathcal{A}^{bh}$ so that $\pi \circ \text{gr } \tilde{Z} = \text{Id}$ (property U: if $\deg D = n$, $\tilde{Z}(\pi(D)) = \pi(D) + (\deg \geq n)$). Hence • π is an isomorphism. • $Z := \tilde{Z} // \pi$ is an expansion.

"God created the knots, all else in topology is the work of mortals."
 Leopold Kronecker (modified) www.katlas.org The Knot Atlas

Action 1.

$\mathcal{A}^{bh} = \mathbb{Q} \langle \dots \rangle$ (then connect using xings or v-xings)

Deriving $\overrightarrow{4T}$. Start from $\overrightarrow{4T} = \dots$ key: use $\times = \times + \times$ (Goussarov-Polyak-Viro) in $\mathcal{I}^n / \mathcal{I}^{n+1}$ using TC \square

Action 2.

The Bracket-Rise Theorem.

Proof.

Corollaries. (1) Related to Lie algebras! (2) Only trees and wheels persist.

Theorem. \mathcal{A}^{bh} is a bi-algebra. The space of its primitives is $FL(T)^H \times CW(T)$, and $\zeta = \log Z$.

ζ is computable! ζ of the Borromean tangle, to degree 5:

I have a nice free-Lie calculator!

Fishy!

Fishy V
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