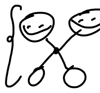
Finite Type Invariants of Ribbon Knotted Balloons and Hoops

Abstract. On my September 17 Geneva talk (ω/sep) I described a certain trees-and-wheels-valued invariant of ribbon knotted loops and 2-spheres in 4-space, and my October 8 Geneva talk (ω/oct) describes its reduction to the Alexander polynomial. Today I will explain how that same invariant arises completely naturally within the theory of finite type invariants of ribbon knotted loops and 2-spheres in 4-space.

Flash



$\zeta : \left\{ \quad \subset \mathbb{R}^4 \right\} = \mathcal{K}^{bh} \longrightarrow \underset{\text{"trees"}}{FL(T)^H} \times \underset{\text{"wheels"}}{CW(T)}$

my goal is to tell you why such invariant is expected, not to derive the formulas.

$\mathcal{K}^{bh} = \mathbb{Q}\left\{ \text{A} \right\} \Big/ \text{relations.}$

Disturbing conjecture: That's all.

$\mathcal{I}^n = \left\{ \substack{\text{pictures} \\ \text{w/ sing.}} \right\} \subset \mathcal{K}^{bh}$

Want "expansion" $Z : \mathcal{K}^{bh} \to \mathcal{A}^{bh} := \bigoplus \mathcal{I}^n/\mathcal{I}^{n+1}$

s.t. if $\gamma \in \mathcal{I}^n$, $Z(\gamma) = (0, \ldots 0, \gamma/\mathcal{I}^{n+1}, *, * \ldots)$

"property U"

Why?

a. Just because, and this is vastly more general.

b. $\left(\mathcal{K}^{bh}/\mathcal{I}^{n+1}\right)^*$ is "finite type/polynomial invariants"

C. The Taylor example.

I'll construct

$$\mathcal{K}^{bh} \xrightarrow{\tilde{Z}} \tilde{\mathcal{A}}^{bh}, \quad \tilde{\mathcal{A}}^{bh} \xrightarrow{\pi} \mathcal{A}^{bh}, \quad \mathcal{K}^{bh} \xrightarrow{Z} \mathcal{A}^{bh} = \bigoplus \mathcal{I}^n/\mathcal{I}^{n+1}$$

1. π is surjective.
2. $\pi /\!/ gr\tilde{Z} = Id$

$\Longrightarrow$ 1. Z is an expansion
2. π is an isomorphism.

Describe $\tilde{\mathcal{A}}^{bh}$

Describe $\tilde{Z}$ [Warning: Not easily computable!]

Bracket rise.

Trees and wheels, $\zeta = \log Z$.

"God created the knots, all else in topology is the work of mortals."

Leopold Kronecker (modified)

www.katlas.org The Knot Atlas

A_e