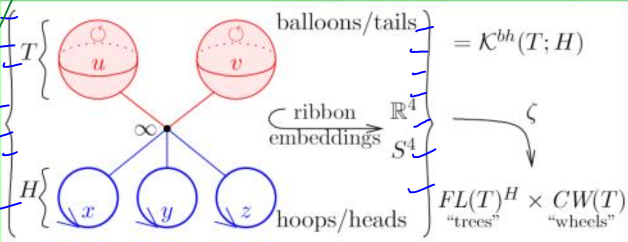
Dror Bar-Natan: Talks: Geneva-131024: ω :=http://www.math.toronto.edu/~drorbn/Talks/Geneva-131024

Finite Type Invariants of Ribbon Knotted Balloons and Hoops

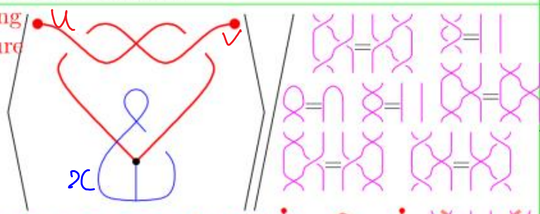
Abstract. On my September 17 Geneva talk (ω/sep) I described a certain trees-and-wheels-valued invariant ζ of ribbon knotted loops and 2-spheres in 4-space, and my October 8 Geneva talk (ω/oct) describes its reduction to the Alexander polynomial. Today I will explain how that same invariant arises completely naturally within the theory of finite type invariants of ribbon knotted loops and 2-spheres in 4-space.



My goal is to tell you why such an invariant is expected, yet not to derive the formulas.

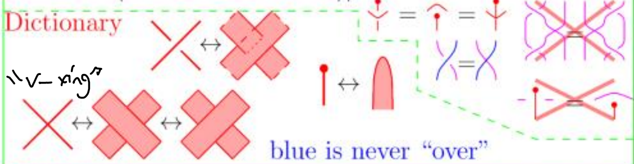
Disturbing Conjecture

$\mathcal{K}^{bh} = \mathbb{Q}\langle$



Dictionary

"v-xing"



Expansions the semi-virtual $:=$ i.e. or

Let $\mathcal{I}^n := \langle$pictures with $\geq n$ semi-virts$\rangle \subset \mathcal{K}^{bh}$.
We seek an "expansion"

$$Z\colon \mathcal{K}^{bh} \to \operatorname{gr}\mathcal{K}^{bh} = \widehat{\bigoplus}\mathcal{I}^n/\mathcal{I}^{n+1} =: \mathcal{A}^{bh}$$

satisfying "property U": if $\gamma \in \mathcal{I}^n$, then

$$Z(\gamma) = (0,\dots,0,\gamma/\mathcal{I}^{n+1},*,*,\dots).$$

Why? • Just because, and this is vastly more general.
• $(\mathcal{K}^{bh}/\mathcal{I}^{n+1})^\star$ is "finite-type/polynomial invariants".
• The Taylor example: Take $\mathcal{K} = C^\infty(\mathbb{R}^n)$, $\mathcal{I} = \{f \in \mathcal{K}\colon f(0)=0\}$. Then $\mathcal{I}^n = \{f\colon f$ vanishes like $|x|^n\}$ so $\mathcal{I}^n/\mathcal{I}^{n+1}$ is homogeneous polynomials of degree n and Z is a "Taylor expansion"! (So Taylor expansions are vastly more general than you'd think).

We'll construct a graded $\tilde{\mathcal{A}}^{bh}$, a surjective graded $\pi\colon \tilde{\mathcal{A}}^{bh} \to \mathcal{A}^{bh}$, and a filtered $\tilde{Z}\colon \mathcal{K}^{bh} \to \mathcal{A}^{bh}$ so that $\pi \,/\!/\, \operatorname{gr}\tilde{Z} = Id$. It follows that • π is an isomorphism. • $Z := \tilde{Z} \,/\!/\, \pi$ is an expansion.

$$\begin{tikzcd} & \tilde{\mathcal{A}}^{bh} \arrow[d, shift left, "\pi"'] \\ \mathcal{K}^{bh} \arrow[ur, "\tilde{Z}"] \arrow[r, "Z"'] & \mathcal{A}^{bh} \arrow[u, shift left, "\operatorname{gr}\tilde{Z}"'] \end{tikzcd}$$

"God created the knots, all else in topology is the work of mortals."
Leopold Kronecker (modified)

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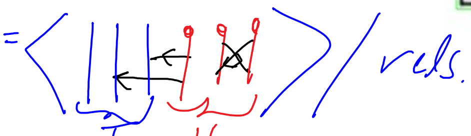
Describe $\mathcal{A}^{bh}$



degree $=$ # of arrows

π : $\to \;\mapsto$ [Then commute using v-xings]

Deriving $4T$: use $Y_1 = X + \cancel{X}$ within $\boxed{\mathbb{B}}$
0 virts: ✓
1 virt: ✓
2 virts: $\vec{6T}$
3 virts:
so in $\mathcal{I}^{*+2}/\mathcal{I}^{*+3}$, $T6T = \pi(6T) = 0$

Describe $\tilde{Z}$ [Warning: Not ~~easily~~ computable!]

easy

Bracket rise. copy from WKO

Trees and wheels, $\zeta = \log Z$.