The next-simplest example

Pavol writes: For the next simplest example look in my paper with David, sect. 3. You can read it, but you can just look at the parenthesized braid with 4 strands which is drawn there. Let \( J \) in \( U(g+g+g+g)[\hbar] \) be what corresponds to this braid.

Now if \( A \) is an algebra in \( U(g+g)-\text{Mod}^\Phi \) (notice: there is no bar over any \( g \)) then:

1. \( A \) is a \( Ug \)-module, where this \( g \) is the diagonal in \( g+g \), but the original product \( m \) on \( A \) is no longer associative in \( Ug-\text{Mod}^\Phi \)

2. however, if we compose \( m \) with the action of \( J \): \( A \otimes A \rightarrow A \otimes A \), then this new product \( m' \) is associative in \( Ug-\text{Mod}^\Phi \)

3. we can have there a spectator Lie algebra \( h \), i.e. \( A \) is in \( U(g+h)-\text{Mod}^\Phi \), and we make it to an associative algebra in \( U(g+h)-\text{Mod}^\Phi \) (with the same \( J \) in \( U(g+g+g+g) \))

There are now two possibilities for the "next simplest" algebra:

1. start with \( A=\text{C}^\infty((G \times \bar{G})/G \times (G \times \bar{G})/G)=\text{C}^\infty(G \times G) \), which is commutative associative in \( U(g+g+\bar{g}+\bar{g}) \), treat \( \bar{g} \) as the spectator \( h \); the composition of the original product with the action of \( J \) makes \( A \) to an associative (but not commutative) algebra in \( U(g+\bar{g}+\bar{g}) \). This is the quantization of the moduli space of a triangle (with one vertex marked with + and two with -)

2. if spectator Lie algebras don’t sound so simple, choose any coisotropic \( C_1, C_2 \subset G \), and start with \( \text{C}^\infty(G/C_1 \times G/C_2) \) as a commutative associative algebra in \( U(g+g)-\text{Mod}^\Phi \).