CUMC Handout on July 6, 2013 (2)

July-06-13
6:19 PM

Dror Bar-Natan: Talks: CUMC-1307:
ω :=http://www.math.toronto.edu/~drorbn/Talks/CUMC-1307

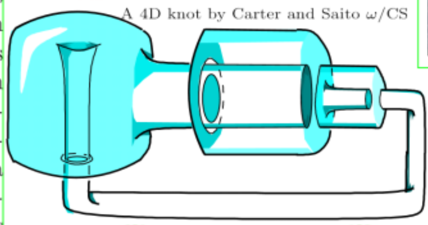
Visualizing the Fourth Dimension

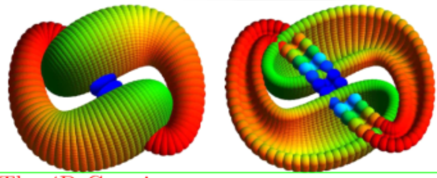
with Ester Dalvit
ω/Dal

Abstract. Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.

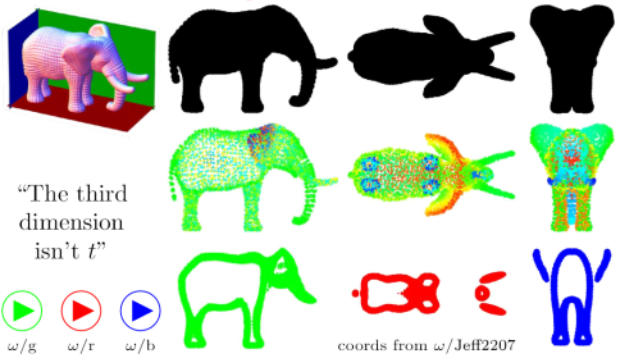
4D Knots.

A 4D knot by Carter and Saito ω/CS

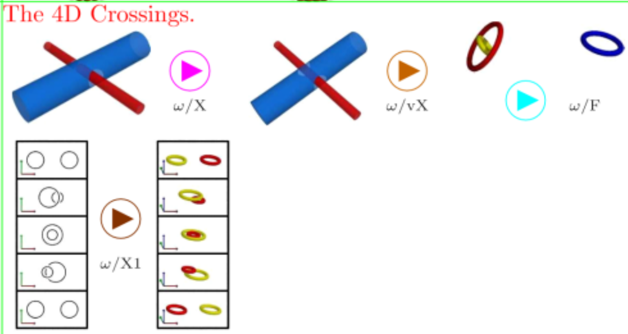




Flatlanders View an Elephant.

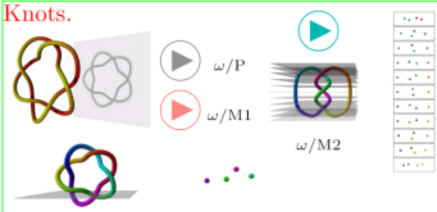


The 4D Crossings.



View as flying rings.
View as coloured tubes.
View as cut tubes
View as "inflated bands".

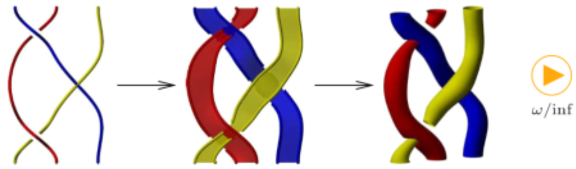
Knots.



Do one knot "right"

1. Project and hide.
2. Project and colour code.
3. Slice and animate.

The Inflation Procedure.



The Crossings.

and the Simplest Thing I Don't Know About It

Satoh's Conjecture. The "kernel" of the "double inflation" map δ, mapping "long" w-knot diagrams in the plane to "long" knotted 2D tubes in 4D, is precisely the moves R1–R3, VR1–VR3, D and OC listed below.

Shin Satoh

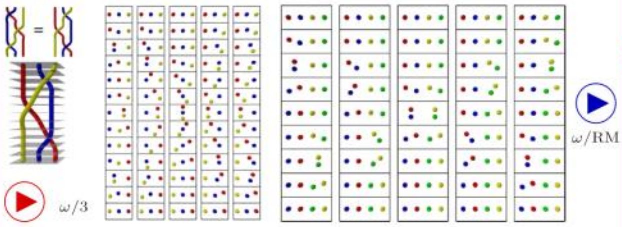
In other words, two long w-knot diagrams represent via δ the same long 2D knotted tube in 4D iff they differ by a sequence of the said moves.

First Iso. Thm: $\phi\colon G \to H \;\Rightarrow\; \operatorname{im}\phi \cong G/\ker(\phi)$

δ is a map from algebra to topology. So a thing in "hard" topology ("ribbon 2-knots") is the same as a thing in "easy" algebra.

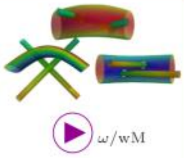
What's "The Same"?

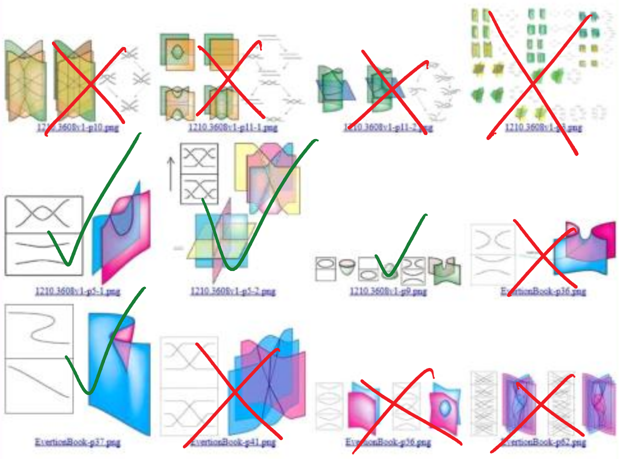
Reidemeister Moves and Theorem.



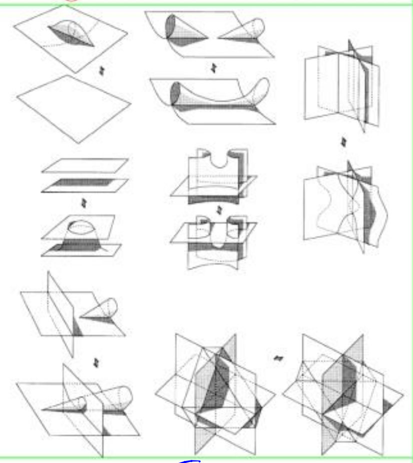
w-Moves.

Carter, Banach, Saito



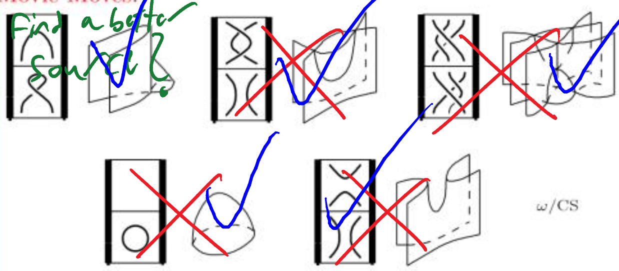


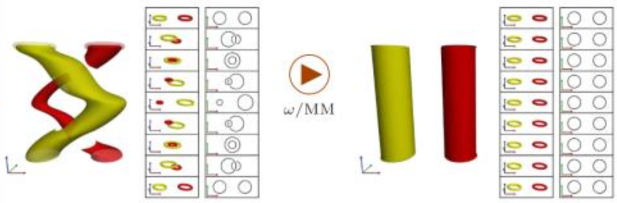
Roseman Moves.

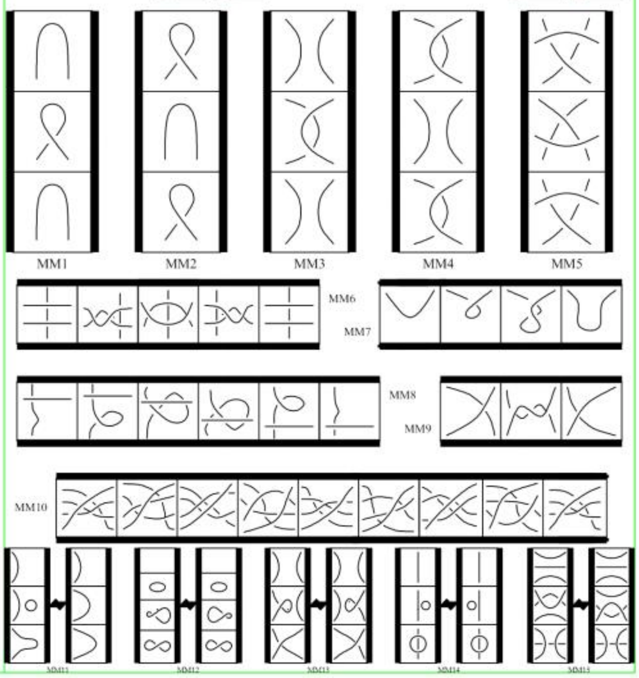


Movie Moves.

Find a better source!







Recycling