

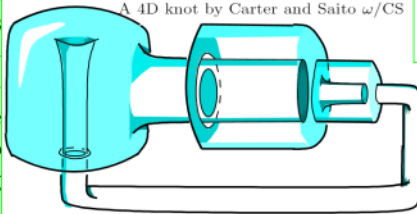
Dror Bar-Natan: Pensieve: 2013-07: Vis4D @ Thursday 4<sup>th</sup> July, 2013  
 $\omega := \text{http://www.math.toronto.edu/~droornb/Talks/CUMC-1307}$

### Visualizing the Fourth Dimension

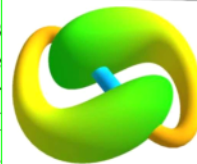
**Abstract.** Much as we can understand 3-dimensional objects by staring at their pictures and x-ray images and slices in 2-dimensions, so can we understand 4-dimensional objects by staring at their pictures and x-ray images and slices in 3-dimensions, capitalizing on the fact that we understand 3-dimensions pretty well. So we will spend some time staring at and understanding various 2-dimensional views of a 3-dimensional elephant, and then even more simply, various 2-dimensional views of some 3-dimensional knots. This achieved, we'll take the leap and visualize some 4-dimensional knots by their various traces in 3-dimensional space, and this achieved, I will tell you about the simplest problem in 4-dimensional knot theory whose solution I don't know.

#### 4D Knots.

A 4D knot by Carter and Saito  $\omega/CS$

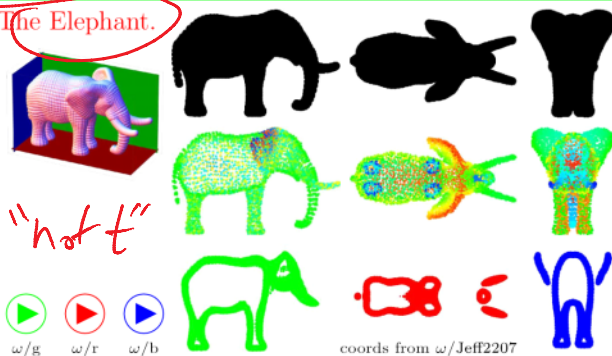


with Ester Dalvit  $\omega/Dal$



Do one knot "right"

#### The Elephant.

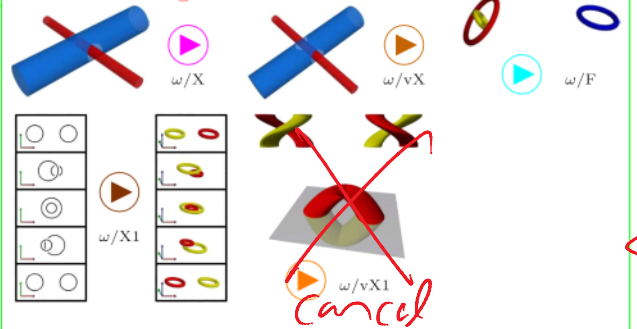


"not t"



coords from  $\omega/Jeff2207$

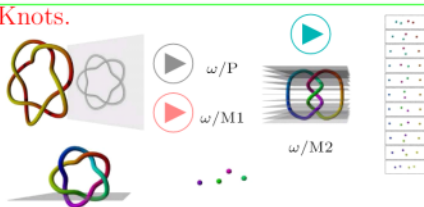
#### The 4D Crossings.



cancel

View as flying rings.  
View as coloured tubes.  
View as cut tubes  
View as "inflated bands".

#### Knots.

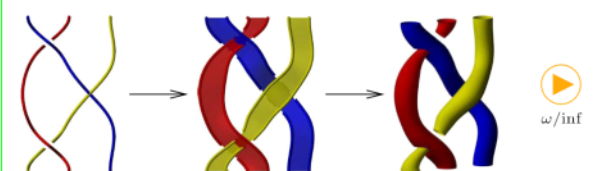


Do one knot "right"

1. Project and hide.
2. Project and colour code.
3. Slice and animate.

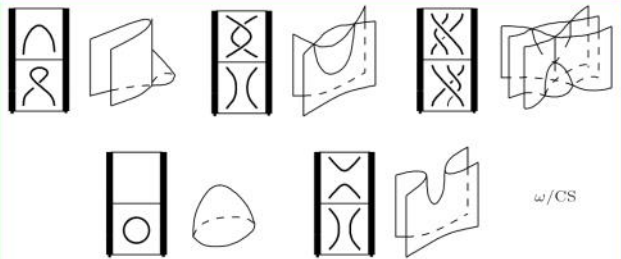
#### The Crossings.

#### The Inflation Procedure.



Satoh's Conjecture.

Satoh's conjecture via double inclusions,  
*write well!*  
 2D → 3D → 4D  
 The diagram below is a complete description  
 of ribbon 2-knots in  $\mathbb{R}^4$ .  
 $\text{---} \text{---} \text{---} = \text{---} \text{---} \text{---}$  etc. What's "The Same"?



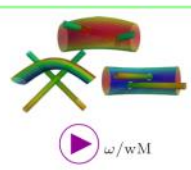
Roseman Moves.



Reidemeister Moves and Theorem.

$\omega/3$   $\omega/RM$

w-Moves.



Movie Moves.

$\omega/MM$