Amitsur-Levitski following Rosset

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(There's also a 7-9 page graph-theoretic proof by Swan)

A-L: If $A_1, \ldots, A_{2n} \in M_n(R)$, where R is a commutative ring, then

$$(*) \quad \sum_{\sigma \in S_{2n}} (-1)^{\sigma} A_{\sigma(1)} \cdots A_{\sigma(2n)} = 0$$

(i) Enough to prove in char 0. (easy)

(ii) Lemma: If R contains $\mathbb{Q}$, $A \in M_n(R)$, $\operatorname{tr} A^i = 0$ for every $i > 0$, then $A^n = 0$

<u>Example</u> $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$

$a_{11} + a_{22} = 0$

$a_{11}a_{11} + a_{12}a_{21} + a_{22}a_{22} + a_{21}a_{12} = 0$

$\widehat{1} + \widehat{2} = 0$

$\widehat{11} + 2\widehat{12} + \widehat{22} = 0 \overset{?}{\Longrightarrow} 111 + 121 = 0$

$\begin{pmatrix} 1 & -2 \\ 1/2 & -1 \end{pmatrix}$ $\quad$ $\begin{pmatrix} 1 & -2 \\ 1/2 & -1 \end{pmatrix}$ $\begin{array}{cc} \begin{pmatrix} 1 & -2 \\ 1/2 & -1 \end{pmatrix} \\ 0 \quad 0 \\ 0 \quad 0 \end{array}$

$\begin{pmatrix} a & b \\ c & -a \end{pmatrix}$ $\quad$ $2a^2 + 2bc = 0$

$bc = -a^2$

$\begin{array}{c} \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \\ \begin{pmatrix} a & b \\ c & -a \end{pmatrix} \begin{matrix} a^2+bc & ab-ba \\ ac-ac & bc-a^2 \end{matrix} \end{array}$

1 $\quad$ 1 $\quad$ 1 $\quad$ 11 $\quad$ 111
$\quad$ 2 $\quad$ 22 $\quad$ 121

2

2 1 2
2 2 2