Knot Theory
Homologies

\[ \frac{3}{2} \text{duality} \]

3D N=2 SUSY

M5-Brane Theory

An Algebraic curve

\[ A^{\text{super}}(x, y, a, t) = 0 \]

\[ x, y \in \mathbb{C}^* \]

\[ \hat{A}^{\text{super}}(x, y, q, a, t) = 0 \]

Chern-Simons Perspective

\[ Z(K, q) = \oint DA \left( Tr_k P e^{\frac{2i}{k} A} \right) e^{\frac{k+i}{2} \text{Tr} (Aa d A + \frac{2}{3} A A A A A) } \]

\[ q = e^L = e^{\frac{2\pi i}{k+N}} N \text{ of SUSY} \]

Related to coloured HOMFLY----

here \( x^n \) is \( q^m \).

Restricts to the Jones polynomial----

Coloured Jones

I'm tired of
\[ \text{Jones}_n(k, q) = J_n(k, a=q^{2k}, q) \]

\[ \frac{1}{n} \exp \left( \frac{1}{n} S_{0}(x) + \frac{1}{n^2} S_{1}(x) + \frac{1}{n^3} S_{2}(x) + \ldots \right) \]

\( q^n = x \text{ fixed} \)

Related to the Volume of the knot "Volume conjecture"

\[ S_{0}(x) = \int \log y \frac{dx}{x} \]

\( A(x, y) = 0 \)

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**Quantum Volume Conjecture**

\[ \text{Jones}_n = 0 \] (relations)

\[ q \text{, } q^2 \text{, } q^3 \text{, } q^4 \text{, } \ldots \]

\[ \text{Jones}_n = q^n \text{ Jones}_n \]

"The AJ conjecture"

\[ \lim_{q \to 1} \hat{A} = A \]

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**Example** For 3_1, \( A(x, y) = (y-1)(y+x^3) \)