Mina Aganagic on Knots and string dualities, Talk 2

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X : Riemannian manifold

QM of a particle of mass 1 on X

$H = \frac{1}{2}\Delta \leftarrow$ Laplace op.

Path integrals on a circle of radius β:

$Z(\beta) = \mathrm{Tr}(e^{-\beta H})$

$\mathcal{H}$ = space of L^2 fncn's on X.

Add Fermions:

$$\mathcal{H} = \mathcal{H}_b \oplus \mathcal{H}_F$$

Add a Fermionic symmetry Q, Q^+

$Q : \mathcal{H}_b \leftrightarrow \mathcal{H}_f \quad Q^2 = Q^{+2} = 0$

$[Q, H] = 0$

$H = \frac{1}{2}(QQ^+ + Q^+Q)$

Typically
$\mathcal{H} = \Omega(X)$
$Q = d \quad Q^+ = *d*$
F = form degree

$\Rightarrow$ bijection (QQ^+) between states in Bosonic & Fermionic Hilbert space, except at energy 0.

except at energy 0.

$I = Tr(-1)^F e^{-\beta H}$ (periodic bndry for F, anti-periodic bndry for B)

$\shortparallel$ Euler char, in typical case $= \sum_p (-1)^p \dim H_p$

$\shortparallel$ $\sum (-1)^p b_p$

$\updownarrow$

contributions come only from ground states — all else, B & F cancel

$P_X(t) = \sum_p t^p b_p \Longleftrightarrow$ Poincare polynomial

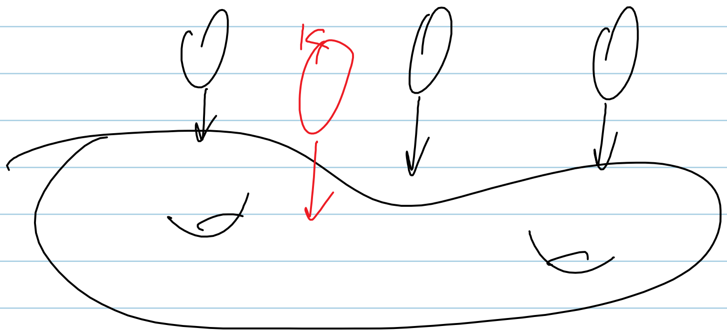
not a partition function!

QM is QFT in $0+1$ dim

0 = space, 1 = time

.... the story generalizes to $d+1$ dims.

Let M be a Seifert 3-manifold



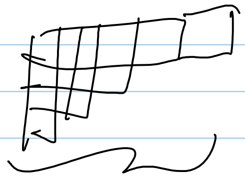
a fibration w/ a semi-free circle action.

May have knots as some of the fibers

..... Example: Torus knots in S^3.

G = ADE group: $SU(N)$; $SO(2N)$; E_N

$\mathcal{H}_{T^2} = WZW(G)$; for $G = SU(N)$

$\mathcal{A}_{T^2} = \Big\{ R =$  $\Big\} N-1 \Big\}$ ....

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.... with completely explicit formulas for S and T.

In math, we used to understand everything we do to the very bottom of things. Let us not lose that!!