Pensieve header: Testing ConciseFastKh.

```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-06"];
<< KnotTheory`
<< ConciseFastKh.m
<< ConciseFastKh-Utilities.m
```

```
Loading KnotTheory` version of February 5, 2013, 3:48:46.4762.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
c1 = Cob[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]], dot[1]]
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]], dot[1]]
```

```
{ECP[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]],
 ECR[S[P[1, 2], P[3, 4]], S[P[2, 3], P[1, 4]]]}
```

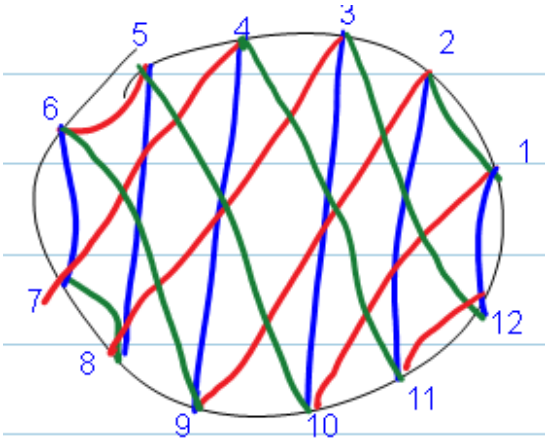
```
{{1 → 1, 2 → 1, 3 → 1, 4 → 1}, ECR[S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]]]}
```

```
{β = S[P[1, 2], P[3, 4]], τ = S[P[2, 3], P[1, 4]]}
```

```
{S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]]}
```

```
{ECP[β, τ], ECR[β, τ]}
```

```
{{1 → 1, 2 → 1, 3 → 1, 4 → 1}, ECR[S[P[1, 2], P[3, 4]], S[P[1, 4], P[2, 3]]]}
```




```
{β = S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 τ = S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
 μ = S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}
```

```
{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 S[P[1, 10], P[2, 9], P[3, 8], P[4, 7], P[5, 6], P[11, 12]],
 S[P[1, 12], P[2, 11], P[3, 10], P[4, 9], P[5, 8], P[6, 7]]}
```

```
{ECP[β, τ], ECP[β, μ], ECP[μ, τ], ECP[β, τ, μ]}
```

```
{{1 → 1, 2 → 1, 3 → 3, 4 → 3, 5 → 1, 6 → 1, 7 → 3, 8 → 3, 9 → 1, 10 → 1, 11 → 3, 12 → 3},
 {1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1},
 {1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1},
 {1 → 1, 2 → 1, 3 → 1, 4 → 1, 5 → 1, 6 → 1, 7 → 1, 8 → 1, 9 → 1, 10 → 1, 11 → 1, 12 → 1}}
```

```
{β /@ Range[4], τ /@ Range[4]}
```

```
{{2, 1, 12, 11}, {10, 9, 8, 7}}
```

```
VCLaw[β, μ, τ]
```

```
{0, {dot[1] → dot[1]}}
```

```
{β, m[4, 11][β], m[1, 5][β]}
```

$$\left\{S[P[1,2],P[3,12],P[4,11],P[5,10],P[6,9],P[7,8]],\ \left\{q\,S[P[1,2],P[3,12],P[5,10],P[6,9],P[7,8]],\ \frac{S[P[1,2],P[3,12],P[5,10],P[6,9],P[7,8]]}{q}\right\},\ \{S[P[2,10],P[3,12],P[4,11],P[6,9],P[7,8]]\}\right\}$$

```
{β, m[4, 11][Q[2] β], m[1, 5][Q[3] β]}
```

```
{S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]],
 m[4, 11][Q[2] S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]]],
 m[1, 5][Q[3] S[P[1, 2], P[3, 12], P[4, 11], P[5, 10], P[6, 9], P[7, 8]]]}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[1]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[2]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[3]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Cob[S[P[1, 2], P[3, 4]], S[P[1, 2], P[3, 4]], dot[4]] // m[2, 3]
```

```
{{dot[1]}}
```

```
Vect[Q[1] S[P[i, j], P[k, l]]]⊗Vect[Q[2] S[P[i, l], P[j, k]]]
```

```
Vect[Q[1] S[P[i, j], P[k, l]]]⊗Vect[Q[2] S[P[i, l], P[j, k]]]
```

```
Kom[{{S[]}}, {}] Cob[S[P[9, 10], P[11, 12]], Q[1] S[P[9, 12], P[10, 11]], 1]
```

```
Cob[S[P[9, 10], P[11, 12]], Q[1] S[P[9, 12], P[10, 11]], 1] Kom[{{S[]}}, {}]
```

```
KhComplex[Knot[3, 1]]
```

```
KnotTheory::loading : Loading precomputed data in PD4Knots`.
```

$$\text{Kom}\left[\left\{\left\{\frac{S[]}{q^9}\right\}\right\},\ \left\{\frac{S[]}{q^5}\right\},\ \{\},\ \left\{\frac{S[]}{q},\ \frac{S[]}{q^3}\right\}\right\},\ \{\{\{0\}\}\},\ 0,\ 0\}\Big]$$

```
KhPoly[Knot[3, 1]]
```

$$\frac{1}{q^3}+\frac{1}{q}+\frac{1}{q^9\,t^3}+\frac{1}{q^5\,t^2}$$

```
Kh[Knot[3, 1]][q, t]
```

KnotTheory::loading : Loading precomputed data in Kh4Knots`.

$$\frac{1}{q^3}+\frac{1}{q}+\frac{1}{q^9\,t^3}+\frac{1}{q^5\,t^2}$$

```
KhPoly[Knot[6, 2]]
```

$$\frac{1}{q^3}+\frac{2}{q}+\frac{1}{q^{11}\,t^4}+\frac{1}{q^9\,t^3}+\frac{1}{q^7\,t^3}+\frac{1}{q^7\,t^2}+\frac{1}{q^5\,t^2}+\frac{1}{q^5\,t}+\frac{1}{q^3\,t}+\frac{t}{q}+q^3\,t^2$$

```
Kh[Knot[6, 2]][q, t]
```

$$\frac{1}{q^3}+\frac{2}{q}+\frac{1}{q^{11}\,t^4}+\frac{1}{q^9\,t^3}+\frac{1}{q^7\,t^3}+\frac{1}{q^7\,t^2}+\frac{1}{q^5\,t^2}+\frac{1}{q^5\,t}+\frac{1}{q^3\,t}+\frac{t}{q}+q^3\,t^2$$

```
KhPoly[Knot[8, 17]] // Timing
```

$$\left\{1.872012,\ \frac{4}{q}+4\,q+\frac{1}{q^9\,t^4}+\frac{2}{q^7\,t^3}+\frac{1}{q^5\,t^3}+\frac{3}{q^5\,t^2}+\right.$$
$$\left.\frac{2}{q^3\,t^2}+\frac{3}{q^3\,t}+\frac{3}{q\,t}+3\,q\,t+3\,q^3\,t+2\,q^3\,t^2+3\,q^5\,t^2+q^5\,t^3+2\,q^7\,t^3+q^9\,t^4\right\}$$

```
Kh[Knot[8, 17]][q, t]
```

$$\frac{4}{q}+4\,q+\frac{1}{q^9\,t^4}+\frac{2}{q^7\,t^3}+\frac{1}{q^5\,t^3}+\frac{3}{q^5\,t^2}+\frac{2}{q^3\,t^2}+$$
$$\frac{3}{q^3\,t}+\frac{3}{q\,t}+3\,q\,t+3\,q^3\,t+2\,q^3\,t^2+3\,q^5\,t^2+q^5\,t^3+2\,q^7\,t^3+q^9\,t^4$$

```
{kh = KhPoly[Knot[8, 21]], kh == Kh[Knot[8, 21]][q, t]} // Timing
```

$$\left\{0.390002,\ \left\{\frac{1}{q^3}+\frac{2}{q}+\frac{1}{q^{15}\,t^6}+\frac{1}{q^{13}\,t^5}+\frac{1}{q^{11}\,t^5}+\right.\right.$$
$$\left.\left.\frac{1}{q^{11}\,t^4}+\frac{1}{q^9\,t^4}+\frac{2}{q^9\,t^3}+\frac{1}{q^7\,t^3}+\frac{1}{q^7\,t^2}+\frac{2}{q^5\,t^2}+\frac{1}{q^5\,t}+\frac{1}{q^3\,t},\ \text{True}\right\}\right\}$$

```
{kh = KhPoly[Knot[10, 165]], kh == Kh[Knot[10, 165]][q, t]} // Timing
```

$$\left\{5.148033,\ \left\{2\,q+q^3+3\,q^3\,t+q^5\,t+3\,q^5\,t^2+3\,q^7\,t^2+3\,q^7\,t^3+3\,q^9\,t^3+4\,q^9\,t^4+\right.\right.$$
$$\left.\left.3\,q^{11}\,t^4+2\,q^{11}\,t^5+4\,q^{13}\,t^5+2\,q^{13}\,t^6+2\,q^{15}\,t^6+q^{15}\,t^7+2\,q^{17}\,t^7+q^{19}\,t^8,\ \text{True}\right\}\right\}$$

```
KhPoly[TorusKnot[6, 5]] // Timing
```

$$\left\{170.134691,\right.$$
$$q^{19}+q^{21}+q^{23}\,t^2+q^{27}\,t^3+q^{25}\,t^4+q^{27}\,t^4+q^{29}\,t^5+q^{31}\,t^5+q^{27}\,t^6+q^{29}\,t^6+q^{31}\,t^7+q^{33}\,t^7+q^{29}\,t^8+$$
$$\left.2\,q^{31}\,t^8+q^{33}\,t^9+2\,q^{35}\,t^9+q^{33}\,t^{10}+2\,q^{37}\,t^{11}+q^{35}\,t^{12}+q^{37}\,t^{12}+q^{41}\,t^{12}+q^{39}\,t^{13}+q^{41}\,t^{13}\right\}$$

```
KhPoly[TorusKnot[9, 5]] // Timing
```

$$\{757.416055,\ q^{31}+q^{33}+q^{35}t^2+q^{39}t^3+q^{37}t^4+q^{39}t^4+q^{41}t^5+q^{43}t^5+q^{39}t^6+q^{41}t^6+q^{43}t^7+q^{45}t^7+q^{41}t^8+2q^{43}t^8+q^{45}t^9+2q^{47}t^9+2q^{45}t^{10}+3q^{49}t^{11}+2q^{47}t^{12}+2q^{49}t^{12}+q^{53}t^{12}+3q^{51}t^{13}+2q^{53}t^{13}+q^{49}t^{14}+2q^{51}t^{14}+q^{55}t^{14}+2q^{53}t^{15}+3q^{55}t^{15}+2q^{53}t^{16}+q^{57}t^{16}+q^{59}t^{16}+3q^{57}t^{17}+q^{55}t^{18}+q^{57}t^{18}+q^{61}t^{18}+2q^{59}t^{19}+q^{61}t^{19}+q^{59}t^{20}+q^{63}t^{20}+q^{63}t^{21}\}$$

```
KhPoly[TorusKnot[7, 6]] // Timing
```

$$\{8555.765644,\ q^{29}+q^{31}+q^{33}t^2+q^{37}t^3+q^{35}t^4+q^{37}t^4+q^{39}t^5+q^{41}t^5+q^{37}t^6+q^{39}t^6+q^{41}t^7+q^{43}t^7+q^{39}t^8+2q^{41}t^8+q^{43}t^9+2q^{45}t^9+q^{41}t^{10}+2q^{43}t^{10}+q^{45}t^{11}+3q^{47}t^{11}+2q^{45}t^{12}+q^{47}t^{12}+q^{51}t^{12}+3q^{49}t^{13}+q^{51}t^{13}+q^{47}t^{14}+q^{49}t^{14}+q^{53}t^{14}+2q^{51}t^{15}+2q^{53}t^{15}+q^{49}t^{16}+q^{51}t^{16}+q^{55}t^{16}+q^{57}t^{16}+q^{53}t^{17}+q^{55}t^{17}+q^{53}t^{18}+q^{57}t^{19}\}$$

```
(Plus @@ (KhPoly[#] == Kh[#][q, t] & /@ AllKnots[{3, 10}])) // Timing
```

{837.241767, 249 True}

```
(Plus @@ (KhPoly[#] == Kh[#][q, t] & /@ AllKnots[11])) // Timing
```

KnotTheory::loading : Loading precomputed data in DTCode4KnotsTo11`.

KnotTheory::credits :

The GaussCode to PD conversion was written by Siddarth Sankaran at the University of Toronto in the summer of 2005.

KnotTheory::loading : Loading precomputed data in Kh4Knots11`.

{6548.734779, 552 True}