Trees and Wheels and Balloons and Hoops

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Abstract. Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and Hoops (KBH) in 4-space behave much like the first and second fundamental groups of a topological space - hoops can be composed like in $\pi_1$, balloons like in $\pi_2$, and hoops "act" on balloons as $\pi_1$ acts on $\pi_2$. We will observe that ordinary knots and tangles in 3-space map into KBH in 4-space and become amalgams of both balloons and hoops.


There's also a paper in progress.

We give an ansatz for a tree and wheel (that is, free-Lie and cyclic word) -valued invariant $\zeta$ of KBHs in terms of the said compositions and action and we explain its relationship with finite type invariants. We speculate that $\zeta$ is a complete evaluation of the BF topological quantum field theory in 4D, though we are not sure what that means. We show that a certain "reduction and repackaging" of $\zeta$ is an "ultimate Alexander invariant" that contains the Alexander polynomial (multivariable, if you wish), has extremely good composition properties, is evaluated in a topologically meaningful way, and is least-wasteful in a computational sense. If you believe in categorification, that's a wonderful playground.

http://www.math.toronto.edu/~dorbn/Talks/NhaTrang-1305/
• Make "universal finite type" more prominent in the handout.
• Put an extra page with "tangle atlas" propaganda?
• Make an extra page for a T&W associator?