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Trees and Wheels and Balloons and Hoops

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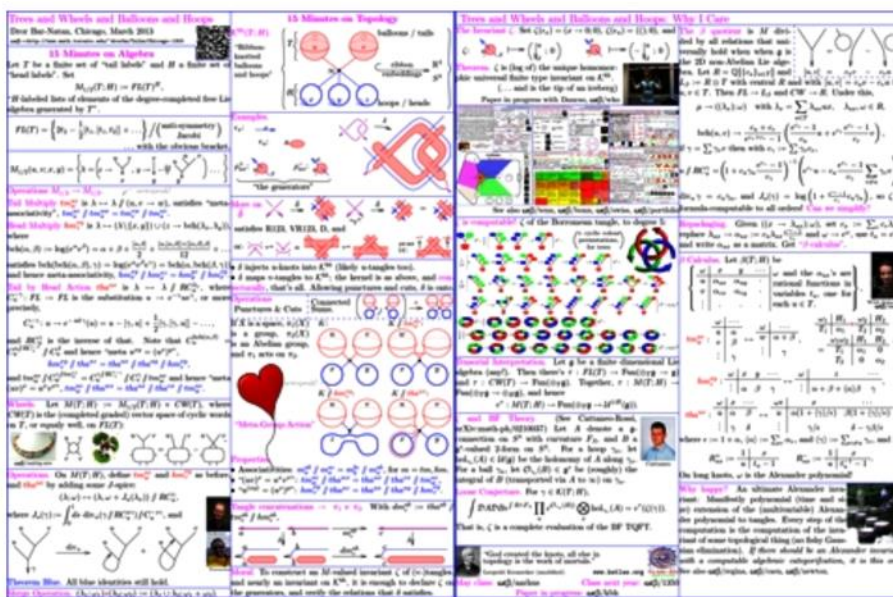
Quantum Topology and Hyperbolic Geometry, Nha Trang, Vietnam, May 2013

Abstract. Balloons are two-dimensional spheres. Hoops are one dimensional loops. Knotted Balloons and Hoops (KBH) in 4-space behave much like the first and second fundamental groups of a topological space - hoops can be composed like in π_1 , balloons like in π_2 , and hoops "act" on balloons as π_1 acts on π_2 . We will observe that

ordinary knots and tangles in 3-space map into KBH in 4-space and become amalgams of both balloons and hoops.

We give an ansatz for a tree and wheel (that is, free-Lie and cyclic word) -valued invariant ζ of KBHs in terms of the said compositions and action and we explain its relationship with finite type invariants. We speculate that ζ is a complete evaluation of the BF topological quantum field theory in 4D, though we are not sure what that means. We show that a certain "reduction and repackaging" of ζ is an "ultimate Alexander invariant" that contains the Alexander polynomial (multivariable, if you wish), has extremely good composition properties, is evaluated in a topologically meaningful way, and is least-wasteful in a computational sense. If you believe in categorification, that's a wonderful playground.

Talk video. 



Handout: [twbh.html](#), [twbh.pdf](#), [twbh.png](#). **Sources:** [twbh.zip](#), [pensieve](#).

There's also a [paper in progress](#).

- Make "universal finite type" more prominent in the handout.
- Put an extra page with "tangle atlas" propaganda?
- Make an extra page for a T&W associator?