Is there a basis-independent version of $J_0$?

Q. If $L_n$ is the $(n-1)!$-dimensional rep. of $S_n$, what's $L_n \otimes_{S_n} L_n^2$?

Is it $(n-1)!$-dimensional w/ basis as above?

By thickening, get a map into $A_n \otimes_{S_n} A_n$: 
I should write the J-story in the language of connections and holonomies!

"multi-deck shuffles"