Juv property - proof

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9:42 AM

$$J_u(s\alpha) + J_v(t\beta /\!/ RC_u^{s\alpha}) /\!/ C_u^{-s\alpha} \stackrel{?}{=} J_v(t\beta) + J_u(s\alpha /\!/ RC_v^{t\beta}) /\!/ C_v^{-t\beta}$$

true at $s=0$, so compute $\frac{d}{ds}$

$$\boxed{(\gamma /\!/ ad_u^{\alpha}) /\!/ RC_u^{\beta} \stackrel{?}{=} (\gamma /\!/ RC_u^{\beta}) /\!/ ad_u^{\alpha /\!/ RC_u^{\gamma}}}$$

$$\alpha /\!/ RC_u^{s\alpha} /\!/ div_u /\!/ C_u^{-s\alpha}$$

$$+ t\beta /\!/ RC_u^{s\alpha} /\!/ ad_u^{\alpha /\!/ RC_u^{s\alpha}} /\!/ \frac{1 - e^{-ad(t\beta /\!/ RC_u^{s\alpha})}}{ad\, t\beta /\!/ RC_u^{s\alpha}} /\!/ RC_v^{t\beta /\!/ RC_u^{s\alpha}} /\!/ div_v$$

$$/\!/ C_v^{-t\beta /\!/ RC_u^{s\alpha}} /\!/ C_u^{-s\alpha}$$

$$- \underbrace{J_v(t\beta /\!/ RC_u^{s\alpha}) /\!/ ad_u^{-\alpha /\!/ RC_u^{s\alpha}}}_{\text{simplify}} /\!/ C_u^{-s\alpha}$$

$$\stackrel{?}{=} \alpha /\!/ \underbrace{RC_v^{t\beta} /\!/ RC_u^{s\alpha /\!/ RC_v^{t\beta}}}_{\text{simplify}} /\!/ div_u /\!/ \underbrace{C_u^{-s\alpha /\!/ RC_v^{t\beta}} /\!/ C_v^{-t\beta}}_{\text{simplify}}$$

may as well take $s=t=1$; also, simplify $J /\!/ ad_u$, and cancel $C_u^{-\alpha}$ on the right

$$\alpha /\!/ RC_u^{\alpha} /\!/ div_u$$

$$+ \beta /\!/ RC_u^{\alpha} /\!/ ad_u^{\alpha /\!/ RC_u^{\alpha}} /\!/ \frac{1 - e^{-ad\, \beta /\!/ RC_u^{\alpha}}}{ad\, \beta /\!/ RC_u^{\alpha}} /\!/ RC_v^{\beta /\!/ RC_u^{\alpha}}$$

$$/\!/ div_v /\!/ C_v^{-\beta /\!/ RC_u^{\alpha}}$$

$$+ [\alpha /\!/ RC_u^{\alpha}, u] /\!/ \frac{1 - e^{-ad\, \beta /\!/ RC_u^{\alpha}}}{ad\, \beta /\!/ RC_u^{\alpha}} /\!/ RC_v^{\beta /\!/ RC_u^{\alpha}}$$

$$/\!/ div_v /\!/ C_v^{-\beta /\!/ RC_u^{\alpha}}$$

$$\stackrel{?}{=} \alpha /\!/ RC_u^{\alpha} /\!/ RC_v^{\beta /\!/ RC_u^{\alpha}} /\!/ div_u /\!/ C_v^{-\beta /\!/ RC_u^{\alpha}}$$

with $\beta /\!/ RC_u^{\alpha} \to \beta$ & $\alpha /\!/ RC_u^{\alpha} \to \alpha$ this is

$$\alpha/\!/\mathrm{div}_u + \beta/\!/\mathrm{ad}_u^{\alpha}/\!/\frac{1-e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}/\!/RC_v^{\beta}/\!/\mathrm{div}_v/\!/C_v^{-\beta}$$

$$+[\alpha,u]/\!/\frac{1-e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}/\!/RC_v^{\beta}/\!/\mathrm{div}_v/\!/C_v^{-\beta}$$

$$\stackrel{?}{=} \alpha/\!/RC_v^{\beta}/\!/\mathrm{div}_u/\!/C_v^{-\beta}$$

Continue in mathematica ....

Attempt on the second derivative:
at $t=0$, this is

$$\alpha/\!/RC_u^{s\alpha}/\!/\mathrm{div}_u/\!/C_u^{-s\alpha} \stackrel{?}{=} \alpha/\!/RC_u^{s\alpha}/\!/\mathrm{div}_u/\!/C_u^{-s\alpha}$$

which holds, so compute $\frac{d}{dt}$:

Aside: $\frac{d}{dt}\left(\frac{1-e^{-at}}{at}\right) = \frac{ae^{-at}\cdot at-(1-e^{-at})a}{a^2t^2}$
$= \frac{-1+e^{-at}(at+a)}{at^2}$

$$\beta/\!/RC_u^{s\alpha}/\!/\mathrm{ad}_u^{\alpha/\!/RC_u^{s\alpha}}/\!/\frac{1-e^{-\mathrm{ad}(t\beta/\!/RC_u^{s\alpha})}}{\mathrm{ad}\,t\beta/\!/RC_u^{s\alpha}}/\!/RC_v^{t\beta/\!/RC_u^{s\alpha}}/\!/\mathrm{div}_v$$
$$/\!/C_v^{-t\beta/\!/RC_u^{s\alpha}}/\!/C_u^{-s\alpha}$$

$+ \ldots$

I need a flat connection on $T^*\oplus FL(T)$
or similar....