Khovanov Homology for Alternating Tangles

**Theorem 1.** If $T$ is non-split alternating, $Kh(T)$ is coherently diagonal.

**Theorem 2.** If $\{\Omega_i\}$ are coherently diagonal and $D$ is alternating planar, then $D(\Omega_1, \Omega_2, \ldots)$ is coherently diagonal.

**Rotation Numbers.** $R(\alpha) := \frac{1}{2} \left( \left| a - h_\alpha \right|_{2k} - \frac{1}{2} \right)$, where $R(\alpha)$ and $R(\gamma) = +1$ and $R(\beta) = -1$.

Also, $R(\alpha \{q\}) := R(\alpha) + q$. Examples:

- $R(\alpha) = \frac{1}{6} - \frac{1}{2} = -\frac{1}{3}$
- $R(\beta) = \frac{1}{2} - \frac{1}{2} = 0$
- $R(\gamma) = \frac{5}{6} - \frac{1}{2} = \frac{1}{3}$

**Alternating Planar Algebra.** All input/output boundaries are connected via the arcs, “in” and “out” strands alternate on all boundaries. A “rotation number” $R_D$ can be defined.

**Proposition 3.2.** $R(D(\sigma_1, \ldots, \sigma_d)) = R_D + \sum_{i=1}^{d} R(\sigma_i)$.

**The Basic Operations.**

- $R_D = +\frac{1}{2}$
- $R_D = 0$

**Diagonal Complexes.** $\Omega: \cdots \to \sigma_j^+ \cdots \to \sigma_j^{-1} \cdots$ such that $2r = R(\sigma_j)$ is a constant $C(\Omega)$.

**Coherently Diagonal Complexes.** All partial closures can be reduced to diagonal, with $C(U(\Omega)) = C(\Omega) - C(D_U)$.

**“Main” Lemma 6.2.** The pairing $D(\Omega_1, \Omega_2)$ via an arc diagram that has at least one boundary arc coming from its first input of a coherently diagonal complex $\Omega_1$ and a diagonal complex $\Omega_2$ is coherently diagonal.

**Gravity and Smoothing.**

**Delooping and Gaussian Elimination.**

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A. have a reduced form which is diagonal.

Add: \( \phi, \theta, \lambda \) are coherently diagonal.

Proof summary is Lemma 6.2 & Thm. 2.