New First Column: 

Let $T$ be a finite set of "tail labels" and $H$ a finite set of "head labels". Set 

$\text{M}_H(T; H) = \text{FL}(T)^H$, the set $H$-labeled lists of elements of the completed free Lie algebra generated by $T$. 

$\text{FL}(T) = \langle t_1, t_2, t_3, t_4, t_5, t_6, t_7, \ldots \rangle / \text{anti-symmetry}$. 

15 Minutes on Algebra
\[ F(\mathbf{T}) = \left\{ 2T_a \pm \frac{1}{2} [T_i, [T_i, T_k]] + \ldots \right\} / \text{anti-symmetry} \]

\[ \text{Jacobi} \]

\[ M_{ij}^{\pm} (x, y; x, y) = \{ (x \rightarrow \frac{y}{x}^\prime; y \rightarrow y^\prime + y^\prime) \} \ldots \}

\text{Operations.}

\[ \text{tm}_{vw}^{uw}: M_{ij}^{uw} \rightarrow M_{ji}^{uw} \text{ by } \lambda \mapsto \lambda \parallel (u, v \rightarrow w) \]

satisfies \[ \text{tm}_{uw}^{vw} / \text{tm}_{vw}^{uw} = \text{tm}_{v}^{uw} / \text{tm}_{w}^{uv} \]

\[ \text{hm}^{x,y}: M_{ij}^{uw} \rightarrow M_{ij}^{uw} \text{ by } \lambda \mapsto (\lambda \{x, y\}) \Leftrightarrow \text{bch}(a, b) \]

where

\[ \text{bch}(a, b) = \log e^a e^b = a + b + \sum_{n=1}^{\infty} \frac{a^nb^n}{n!} + \ldots \]

satisfies \[ \text{bch} \left( \text{bch}(a, b), c \right) = \text{bch}(a, \text{bch}(b, c)) \]

hence \[ \text{hm}^{x, y} / \text{hm}^{x, z} = \text{hm}^{y, z} / \text{hm}^{x, y} \]

\[ \text{tma}_{uw}^{uw}: M_{ij}^{uw} \rightarrow M_{ij}^{uw} \text{ by } \lambda \mapsto \lambda / \text{RC}_{uw} \]

where

\[ \text{C}_{u}: F \rightarrow F \text{ is } u \mapsto e^{-a(x)}(u) = \text{write} \]

and \[ \text{RC}_{u} \] is the inverse of that. \[ \text{C} \]

\[ \text{satisfies} \]

\[ \text{C}^{\text{bch}(a, b)} = \text{C}^{\text{bch}(a, b)} / \text{C}^{\text{bch}(a, b)} \]

hence \[ \text{b-action axiom} \]

and

\[ \text{tm}_{uw}^{uw} / \text{C}_{u} = \text{C}_{u} / \text{tm}_{uw}^{uw} \]

hence \[ \text{t-action axiom} \]

Consider \[ \text{C}_{u}^{\pm \theta}, \text{RC}_{u}^{\pm \theta} \]
Trees and Wheels and Balloons and Hoops and Why I Care

The invariant ω. Set ω(π²) = (±ω², 0). This at least defines an invariant of u/v/w-tangles, and if the topologists will deliver a "hands-free" theorem, it is well-defined on K³.4

Theorem. ω is (the log of a) universal finite-type invariant (a homomorphic expansion) of w-tangles.

General Interpretation. Let g be a finite-dimensional Lie algebra (any?). Then there is a function T ↦ Fun(R[R] → g). Together, ω : M(T, H) → Fun(123 ↦ g), and hence

Case 1: M(T, H) → Fun((y, z) → exp(αy, z)).

Case 2: M(T, H) → Fun((y, z) → exp(αy, z)).

Infinite Conjecture. For γ ∈ K(T, H),

\[ \int DADB \sum_{\beta} Tr_{\alpha}(\beta) K_{\alpha}(\beta) | \omega(\beta) = \epsilon(\gamma) | \]

That is, ω is a complete evaluation of the BF TQFT. Issues: How exactly is B transported via A to 3? How does the ribbon condition arise? Or if it doesn’t, could it be that ω can be generalized?5

The invariant ω. Arises when g is the 2D non-Abelian Lie algebra.

Arises when reducing by relations satisfied by the weight system of the Alexander polynomial.

"Confined to the knots, all the m-tangles are the knots of knots."

Paper in progress. www.xkcd.com

The β quotient. Let R = Q[⟨x₁, x₂⟩] and I₂ := R × T with control R and with [u, v] = c₁u + c₂v for u, v ∈ T. The FL = I₂ and CW → R. Under this,

\[ \mu = \langle (a, b) \rangle \text{ with } a = \sum_{\lambda \in R, \omega \in R} \lambda \text{ and } b \in R. \]

If λ = \sum cₙλₙ then with cₙ = \sum λₙₙ, then

\[ \mu = \left( \sum_{\lambda \in R, \omega \in R} \lambda \text{ and } b \in R \right) \]

If \( R_{\omega_{\lambda}} = \left\{ x \in R \right\} \]

Then \( R_{\omega_{\lambda}} = \left\{ x \in R \right\} \]

By this, \( R_{\omega_{\lambda}} = \left\{ x \in R \right\} \]

So \( R_{\omega_{\lambda}} = \left\{ x \in R \right\} \]

Can we simplify? Reorganizing. Given \( \langle x : \lambda \rangle \), we set \( c_{\lambda} = \sum_{\lambda \in R} c_{\lambda} \theta_{\lambda} \) and replace \( \lambda \) with \( \lambda \). Let \( \lambda \) with \( \lambda \). Let \( \lambda \) with \( \lambda \).

Get \( \omega \): calculus. Let \( \omega(R, T) \) be

\[ \omega : x \to y \to \cdots \omega \text{ and the } \alpha_{R,T} \text{ are rational functions in variables } \omega_{\alpha_{R,T}} \text{ one for each } u \in T. \]

Then

\[ \omega_{\alpha_{R,T}} = \frac{1}{\omega} \left( \frac{H_1}{\omega_{R,T}} \right) \]

\[ \omega_{\alpha_{R,T}} = \frac{1}{\omega} \left( \frac{H_1}{\omega_{R,T}} \right) \]

\[ \omega_{\alpha_{R,T}} = \frac{1}{\omega} \left( \frac{H_1}{\omega_{R,T}} \right) \]

\[ \omega_{\alpha_{R,T}} = \frac{1}{\omega} \left( \frac{H_1}{\omega_{R,T}} \right) \]

\[ \omega_{\alpha_{R,T}} = \frac{1}{\omega} \left( \frac{H_1}{\omega_{R,T}} \right) \]

On long knots, \( \omega \) is the Alexander polynomial!

Why bother? (1) An ultimate Alexander invaraint: Manifestly polynomial (time and extension) expression of the (multivariable) Alexander polynomial to tangles. Every step of the computation is the computation of the invariant of some topological thing (no fancy Gaussian elimination). If there should be an Alexander invariant to have algebraic categorification, it is this one.

See also waltz, waltz, waltz.

Why bother? (2) Related to A, T, K, and E, \( K \) should have vast generalization beyond w knots and the Alexander polynomial.

See also waltz, waltz, waltz.
Trees and Wheels and Balloons and Hoops — Extras / Recycling

Invariant #6. With $\Pi_1$ denoting “homotopy class,” map $\gamma \in \pi_1([a,b],[c,d])$, where the meridians of the balls $a_i$ normally generate $\Pi_1$, and the “longitudes” $x_j$ are some elements of $\Pi_1$.

Note like $\cdot$, $\cdot$ acts by “merging” two meridians/generators. $\alpha \cdot m$ acts by multiplying two longitudes, and $\text{th} m^n$ acts by “conjugating a meridian by a longitude.”

$(\Pi, (\ldots), (x, \ldots)) \mapsto (\Pi \times \mathbb{R}/(\alpha = x \cdot e^{-1})), (\ldots), (x, \ldots)$

Failure #6 Can we write the $x$'s as free words in the n'x's?

If $x = wv$, compute $x \cdot \text{th} wv$:

$z = uv \rightarrow \text{th} v = y^x v = y^x y^x v = y^x y^x y^x v = \ldots$

Why ODE? Q: Find f s.t. $f(x+y) = f(x)f(y)$

A. $\frac{df}{dt} = \frac{d}{dx}(x + y) = \frac{d}{dx} f(x) f(y) = f(y) f(x)$

Now solve this ODE using Picard's theorem or power series.

Scheme:

- Balloons and hoops in $\mathbb{R}^4$, algebraic structure and relations with 3D.
- An ansatz for a “homomorphic” invariant: computable related to finite-type and to BF.
- Reduction to an “ultimate Alexander invariant”.