Cheat Sheet J on March 25, 8:29PM

March-26-13
10:49 AM

Cheat Sheet J

http://drorbn.net/AcademicPensieve/2013-03/
initiated 18/3/13; completed ?; modified 25/3/13, 8:29pm

With alphabet T and with $u,v,w\in T$, $\alpha,\beta,\gamma\in FL(T)$, $D\in\mathfrak{tder}(T)$, $g,h\in\exp(\mathfrak{tder}(T))=\mathrm{TAut}(T)$. Checkmarks (✓) as in `CheatSheetJ-Verification.nb`.

---

1. The definition of J: $J_u(\gamma):=\int_0^1 ds\,\mathrm{div}_u(\gamma\,/\!\!/\,RC_u^{s\gamma})\,/\!\!/\,C_u^{-s\gamma}$
2. ✓ The t equation (desired):
$$J_w(\gamma\,/\!\!/\,tm_w^{uv})\,/\!\!/\,RC_w^{\gamma/\!\!/tm_w^{uv}}=J_u(\gamma)\,/\!\!/\,tm_w^{uv}\,/\!\!/\,RC_w^{\gamma/\!\!/tm_w^{uv}}+J_v(\gamma\,/\!\!/\,RC_u^{\gamma})\,/\!\!/\,RC_v^{\gamma/\!\!/RC_u^{\gamma}}\,/\!\!/\,tm_w^{uv}$$
3. ✓ The h equation (desired): $J_u(\mathrm{bch}(\alpha,\beta))=J_u(\alpha)+J_u(\beta\,/\!\!/\,RC_u^{\alpha})\,/\!\!/\,C_u^{-\alpha}$

---

4. RC equation t: $tm_w^{uv}\,/\!\!/\,RC_w^{\gamma/\!\!/tm_w^{uv}}=RC_u^{\gamma}\,/\!\!/\,RC_v^{\gamma/\!\!/RC_u^{\gamma}}\,/\!\!/\,tm_w^{uv}$
5. RC equation h: $RC_u^{\mathrm{bch}(\alpha,\beta)}=RC_u^{\alpha}\,/\!\!/\,RC_u^{\beta/\!\!/RC_u^{\alpha}}$
6. RCC equation div: $\mathrm{div}_u(\alpha\,/\!\!/\,RC_u^{\gamma})\,/\!\!/\,C_u^{\gamma}=?$
7. CRC equation div: $\mathrm{div}_u(\alpha\,/\!\!/\,C_u^{\gamma})\,/\!\!/\,RC_u^{\gamma}=?$

---

8. div property t: $\mathrm{div}_w(\gamma\,/\!\!/\,tm_w^{uv})=(\mathrm{div}_u(\gamma)+\mathrm{div}_v(\gamma))\,/\!\!/\,tm_w^{uv}$
9. ✓ div property h — the "cocycle condition": with $\mathrm{ad}_u\{\gamma\}:=\mathrm{der}(u\to[\gamma,u])$,
$$(\mathrm{div}_u\,\alpha)\,/\!\!/\,\mathrm{ad}_u\{\beta\}-(\mathrm{div}_u\,\beta)\,/\!\!/\,\mathrm{ad}_u\{\alpha\}=\mathrm{div}_u\left([\alpha,\beta]+\alpha\,/\!\!/\,\mathrm{ad}_u\{\beta\}-\beta\,/\!\!/\,\mathrm{ad}_u\{\alpha\}\right)$$
10. div of bch: $\mathrm{div}_u(\mathrm{bch}(\alpha,\beta))=?$

---

11. The definition of JA: $JA_u(\gamma):=J_u(\gamma)\,/\!\!/\,RC_u^{\gamma}$
12. The ODE for JA: with $\gamma_s=\gamma\,/\!\!/\,RC_u^{s\gamma}$,
$$JA(0)=0,\qquad\frac{dJA(s)}{ds}=JA(s)\,/\!\!/\,\mathrm{ad}_u\{\gamma_s\}+\mathrm{div}_u\,\gamma_s,\qquad JA(1)=JA_u(\gamma)$$

---

13. The relation with $\mathfrak{tder}$: $e^{\mathrm{ad}_u\{\gamma\}}=C_u^{?}$ and $C_u^{\gamma}=e^{\mathrm{ad}_u\{?\}}$
14. The definition of j (following A-T): $j(e^D)=\int_0^1 ds\,e^{sD}(\mathrm{div}\,D)=\frac{e^D-1}{D}(\mathrm{div}\,D)$
15. j's cocycle property: $j(gh)=j(g)+g\cdot j(h)$

---

16. The differential of exp: $\delta e^{\gamma}=e^{\gamma}\cdot\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma)=\left(\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right)(\delta\gamma)\cdot e^{\gamma}$
17. ✓ The differential of $\gamma=\mathrm{bch}(\alpha,\beta)$:
$$\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma)=\left(e^{-\mathrm{ad}\,\beta}\frac{1-e^{-\mathrm{ad}\,\alpha}}{\mathrm{ad}\,\alpha}\right)(\delta\alpha)+\left(\frac{1-e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}\right)(\delta\beta)$$
18. ✓ The differential of C: $\delta C_u^{\gamma}=\mathrm{ad}_u\left\{\left(\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right)(\delta\gamma)\,/\!\!/\,RC_u^{-\gamma}\right\}\,/\!\!/\,C_u^{\gamma}$
19. ✓ The differential of RC: $\delta RC_u^{\gamma}=RC_u^{\gamma}\,/\!\!/\,\mathrm{ad}_u\left\{\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma)\,/\!\!/\,RC_u^{\gamma}\right\}$

---

20. The differential of J: $\delta J_u(\gamma)=?$

Add something about $h\Delta$, $t\Delta$ and their respective equations?

$$\left.\begin{array}{l} t\Delta^w_{uv}\,/\!\!/\,tha^{ux}\,/\!\!/\,tha^{vx}=tha^{wx}\,/\!\!/\,t\Delta^w_{uv}\ ? \\ h\Delta^z_{xy}\,/\!\!/\,tha^{ux}=h\Delta^z_{xy}\,/\!\!/\,tha^{uy}=tha^{uz}\,/\!\!/\,h\Delta^z_{xy}\ ? \end{array}\right\}\text{Seems easy}$$

$hA^z_{xy} // tha^{ux} = hA^z_{xy} // tha^{uy} = tha^{uz} // hA^z_{xy}$?} easy

---

$$\delta J_u = \int_0^1 ds \Big[div_u(\delta\gamma // RC_u^{s\gamma}) // C_u^{-s\gamma}$$

might these cancel?

$$+ s\, div_u\Big(\underbrace{\gamma // RC_u^{s\gamma}}_{\alpha} // ad_u^{\overbrace{\frac{1-e^{-ads\gamma}}{ads\gamma}(\delta\gamma)//RC_u^{s\gamma}}^{\beta}}\Big) // C_u^{-s\gamma}$$

$$- s\, div_u(\gamma // RC_u^{s\gamma}) // ad_u^{\left(\frac{1-e^{-ads\gamma}}{ads\gamma}\right)(\delta\gamma)//RC_u^{s\gamma}} // C_u^{-s\gamma} \Big]$$

Clearly, I should compare

$$div_u(\alpha // ad_u^{\frac{1-e^{-ad\beta}}{ad\beta}(\gamma)//RC_u^\beta})$$

with $\quad div_u(\alpha) // ad_u^{\frac{1-e^{-ad\beta}}{ad\beta}(\gamma)//RC_u^\beta}$

perhaps simply

$$div_u(\alpha // ad_u^\beta) \quad \text{with} \quad div_u(\alpha) // ad_u^\beta$$

the cocycle eqn' for div is about that.

$$div(\alpha // ad_u\beta) - div_u(\alpha) // ad_u^\beta$$
$$= div_u(\beta // ad_u^\alpha) - div_u(\beta) // ad_u^\alpha - div_u[\alpha,\beta]$$

So

$$\delta J_u = \int_0^1 ds\, div_u(\delta\gamma // RC_u^{s\gamma}) // C_u^{-s\gamma}$$

$$+ \int_0^1 ds \Big[div_u\left(\frac{1-e^{-ads\gamma}}{ad\gamma}(\delta\gamma) // RC_u^{s\gamma} // ad_u^{\gamma // RC_u^{s\gamma}}\right)$$

$$- div_u\left(\frac{1-e^{-ads\gamma}}{ad\gamma}(\delta\gamma) // RC_u^{s\gamma}\right) // ad_u^{\gamma // RC_u^{s\gamma}}$$

$$- \operatorname{div}_u\left(\left(1-e^{-\operatorname{ad} s\gamma}\right)(\delta\gamma) // RC_u^{s\gamma}\right] // C_u^{-s\gamma}$$

$$= \int_0^1 ds \left[\operatorname{div}_u\left(\left(e^{-\operatorname{ad} s\gamma}(\delta\gamma) + \frac{1-e^{-\operatorname{ad} s\gamma}}{\operatorname{ad}\gamma}(\delta\gamma)\right) // \operatorname{ad}_u^{\gamma}\right) // RC_u^{s\gamma}\right)$$

rough $$- \operatorname{div}_u\left(\frac{1-e^{-\operatorname{ad} s\gamma}}{\operatorname{ad}\gamma}(\delta\gamma) // RC_u^{s\gamma}\right) // \operatorname{ad}_u^{\gamma // RC_u^{s\gamma}} \Big] // C_u^{-s\gamma}$$

Aside: Can I simplify the last term?

$$\operatorname{div}_u\left(\frac{1-e^{-\operatorname{ad}\alpha}}{\operatorname{ad}\alpha}(\beta) // RC_u^{\alpha}\right) // \operatorname{ad}_u^{\alpha // RC_u^{\alpha}} // C_u^{-\alpha}$$

$$= \operatorname{div}_u\left(\frac{1-e^{-\operatorname{ad}\alpha}}{\operatorname{ad}\alpha}(\beta) // RC_u^{\alpha}\right) // C_u^{-\alpha} // \operatorname{ad}_u^{\alpha}$$