Cheat Sheet J on March 20, 7:11AM

March-20-13
9:06 AM

Change notation to $\mathrm{ad}_u\{\gamma\} = \mathrm{der}(u \to [\gamma, u])$

A: with an alphabet T, $u, v, w \in T$, $\alpha, \beta, \gamma \in FL(T)$, $D \in \mathrm{tder}(T)$, $g, h \in \exp(\mathrm{tder}(T)) = \mathrm{TAut}(T)$

\tbd = {\red ?}

A

Cheat Sheet J

\tbd

http://drorbn.net/AcademicPensieve/2013-03/
initiated 18/3/13; completed ???; modified 20/3/13, 7:11am

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1. The definition of J: $$J_u(\gamma) := \int_0^1 ds\ \mathrm{div}_u(\gamma /\!\!/ RC_u^{s\gamma}) /\!\!/ C_u^{-s\gamma}$$
2. The t equation (desired): $$J_w(\gamma /\!\!/ tm_w^{uv}) /\!\!/ RC_w^{\gamma /\!\!/ tm_w^{uv}} = J_u(\gamma) /\!\!/ tm_w^{uv} /\!\!/ RC_w^{\gamma /\!\!/ tm_w^{uv}} + J_v(\gamma /\!\!/ RC_u^{\gamma}) /\!\!/ RC_v^{\gamma /\!\!/ RC_u^{\gamma}} /\!\!/ tm_w^{uv}$$
3. The h equation (desired): $$J_u(\mathrm{bch}(\alpha, \beta)) = J_u(\alpha) + J_u(\beta /\!\!/ RC_u^{\alpha}) /\!\!/ C_u^{-\alpha}$$

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4. CRC equation t: $$tm_w^{uv} /\!\!/ RC_w^{\gamma /\!\!/ tm_w^{uv}} = RC_u^{\gamma} /\!\!/ RC_v^{\gamma /\!\!/ RC_u^{\gamma}} /\!\!/ tm_w^{uv}$$
5. CRC equation h: $$RC_u^{\mathrm{bch}(\alpha,\beta)} = RC_u^{\alpha} /\!\!/ RC_u^{\beta /\!\!/ RC_u^{\alpha}}$$
6. CRC equation div: $$\mathrm{div}_u(\alpha /\!\!/ RC_u^{\beta}) /\!\!/ C_u^{\beta} = \text{\tbd}$$
   γ γ

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7. div property t: $$\mathrm{div}_w(\gamma /\!\!/ tm_w^{uv}) = (\mathrm{div}_u(\gamma) + \mathrm{div}_v(\gamma)) /\!\!/ tm_w^{uv}$$
8. div property h — the "cocycle condition": with $\mathrm{ad}_u^{\gamma} := \mathrm{der}(u \to [\gamma, u])$, $$(\mathrm{div}_u \beta) /\!\!/ \mathrm{ad}_u^{\alpha} + (\mathrm{div}_u \alpha) /\!\!/ \mathrm{ad}_u^{\beta} = \mathrm{div}_u([\beta, \alpha] + \mathrm{ad}_u^{\alpha}(\beta) - \mathrm{ad}_u^{\beta}(\alpha))$$

C

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9. The definition of JA: $$JA_u(\gamma) := J_u(\gamma) /\!\!/ RC_u^{\gamma}$$
10. The ODE for JA: with $\gamma_s = \gamma /\!\!/ RC_u^{s\gamma}$, $$JA(0) = 0, \quad \frac{dJA(s)}{ds} = JA(s) /\!\!/ \mathrm{ad}_u^{\gamma_s} + \mathrm{div}_u \gamma_s, \quad JA(1) = JA_u(\gamma)$$

B

11. The definition of j (following A-T): $$j(e^D) = \int_0^1 ds\, e^{sD}(\mathrm{div}\ D) = \frac{e^D - 1}{D}(\mathrm{div}\ D)$$
12. j's cocycle property: $$j(gh) = j(g) + g \cdot j(h)$$

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13. The differential of exp: $$\delta e^{\gamma} = e^{\gamma} \cdot \left(\frac{1 - e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) = \left(\frac{e^{\mathrm{ad}\,\gamma} - 1}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) \cdot e^{\gamma}$$
14. The differential of $\gamma = \mathrm{bch}(\alpha, \beta)$: $$\left(\frac{1 - e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) = \left(e^{-\mathrm{ad}\,\beta}\frac{1 - e^{-\mathrm{ad}\,\alpha}}{\mathrm{ad}\,\alpha}\right)(\delta\alpha) + \left(\frac{1 - e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}\right)(\delta\beta)$$
15. The differential of C (approximately): $$\delta C_u^{\gamma} = \mathrm{ad}_u^{\left(\frac{1 - e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) /\!\!/ RC_u^{-\gamma}} /\!\!/ C_u^{\gamma}$$
16. The differential of RC: $$\delta RC_u^{\gamma} =$$

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17. The differential of J: $$\delta J_u(\gamma) = \text{\tbd}$$

B: $D = ad_u\{\gamma\}$ $\quad e^D(\alpha) = \alpha // C_u^{\overset{?}{1}}$

C: div of bch: $\quad \mathrm{div}_u(bch(\alpha,\beta)) \overset{?}{=}$

D: $\delta RC_u^\gamma = RC_u^\gamma // ad_u\left\{\left(\frac{e^{ad\gamma}-1}{ad\gamma}(\delta\gamma)\right) // RC_u^\gamma\right\}$