

Pensieve header: Trees and Wheels for the Borromean Rings.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2013-03"] ;
<< FreeLie.m

Domain[f_List] := First /@ f;
f_ \ key_ := DeleteCases[f, key → _];
f_ \ keys_List := Fold[#, #2 &, f, keys];
f1_List ≡ f2_List := Domain[f1] === Domain[f2] && (And @@ (
    (# /. f1) ≡ (# /. f2)) & /@ Domain[f1]
));
(* LieDerivation[der_][f_List] := MapAt[LieDerivation[der], f, {All, 2}]; *)
LieMorphism[mor_][f_List] := MapAt[LieMorphism[mor], f, {All, 2}];
M /: M[λ1_, w1_] ∪ M[λ2_, w2_] := M[λ1 ∪ λ2, w1 + w2];
M[λ1_, w1_] ≡ M[λ2_, w2_] := (λ1 ≡ λ2) && (w1 ≡ w2);

tm[u_, v_, w_][λ_List] := λ // LieMorphism[⟨u⟩ → ⟨w⟩, ⟨v⟩ → ⟨w⟩];
tm[u_, v_, w_][M[λ_, w_]] := LieMorphism[⟨u⟩ → ⟨w⟩, ⟨v⟩ → ⟨w⟩] /@ M[λ, w];
hm[x_, y_, z_][λ_List] := Union[λ \ {x, y}, {z → BCH[x/. λ, y/. λ]}];
hm[x_, y_, z_][M[λ_, w_]] := M[λ // hm[x, y, z], w];
RC[u_, λx_LieSeries, ub_][ser_] := StableApply[
    LieMorphism[⟨u⟩ → Ad[λx][⟨ub⟩]],
    ser
];
RC[u_, λx_LieSeries][ser_] :=
    ser // RC[⟨u⟩, λx, ⟨"v"⟩] // LieMorphism[⟨"v"⟩ → ⟨u⟩];
J[u_, λx_] := Module[{s},
    IntegrateCWSeries[
        div[⟨u⟩, λx // RC[⟨u⟩, s λx]] // LieMorphism[⟨u⟩ → Ad[-s λx][⟨u⟩]],
        {s, 0, 1}
    ]
];
tha[u_, x_][λ_List] := MapAt[RC[⟨u⟩, x /. λ], λ, {All, 2}];
tha[u_, x_][M[λ_, w_]] :=
    M[λ // tha[u, x], (w + J[⟨u⟩, x /. λ]) // RC[⟨u⟩, x /. λ]];
dm[a_, b_, c_][μ_] := μ // tha[⟨a⟩, b] // tm[⟨a⟩, ⟨b⟩, ⟨c⟩] // hm[a, b, c];
R+[u_, x_] := M[{x → MakeLieSeries[⟨u⟩]}, u → MakeLieSeries[0]], MakeCWSeries[0]];
R-[u_, x_] := M[{x → MakeLieSeries[-⟨u⟩]}, u → MakeLieSeries[0]], MakeCWSeries[0]];

μ0 = R-[r, 6] ∪ R+[2, 4] ∪ R-[g, 9] ∪ R+[5, 7] ∪ R-[b, 3] ∪ R+[8, 1]

M[{1 → LS[8, 0, 0], 2 → LS[0, 0, 0], 3 → LS[-b, 0, 0], 4 → LS[2, 0, 0],
    5 → LS[0, 0, 0], 6 → LS[-r, 0, 0], 7 → LS[5, 0, 0], 8 → LS[0, 0, 0],
    9 → LS[-g, 0, 0], b → LS[0, 0, 0], g → LS[0, 0, 0], r → LS[0, 0, 0}], CWS[0, 0, 0]]

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Do[μ0 = μ0 // dm[r, k, r], {k, 1, 3}];
Do[μ0 = μ0 // dm[g, k, g], {k, 4, 6}];
Do[μ0 = μ0 // dm[b, k, b], {k, 7, 9}];
μ0

M[ {b → LS[0, gr, 1/2 gg r + br g + 1/2 g rr], g → LS[0, -br, 1/2 b br - bg r - br g + 1/2 b rr],
r → LS[0, bg, 1/2 bb g + bg r + 1/2 bgg]}, CWS[0, 0, 2 CW[bgr]]]

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Trees

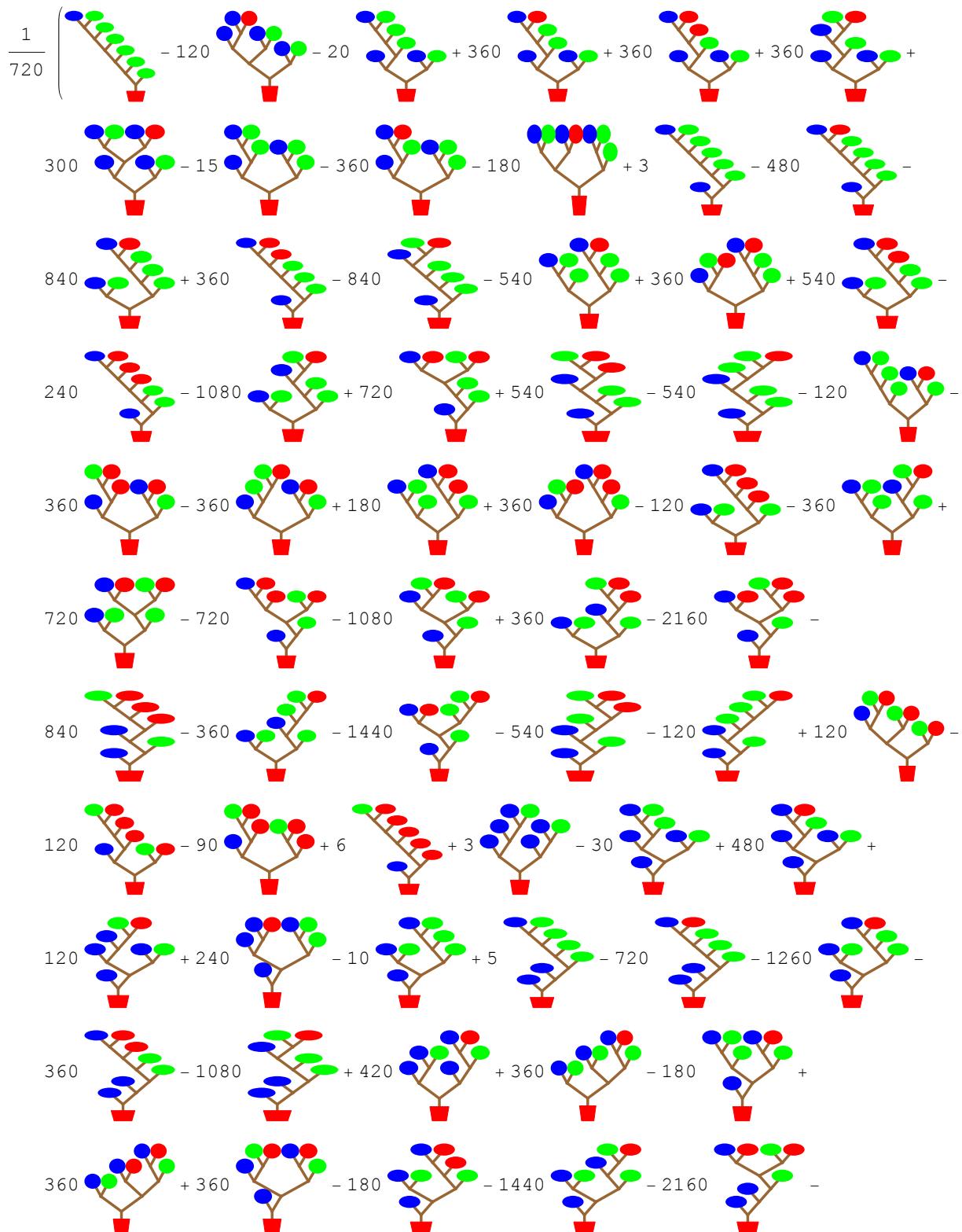
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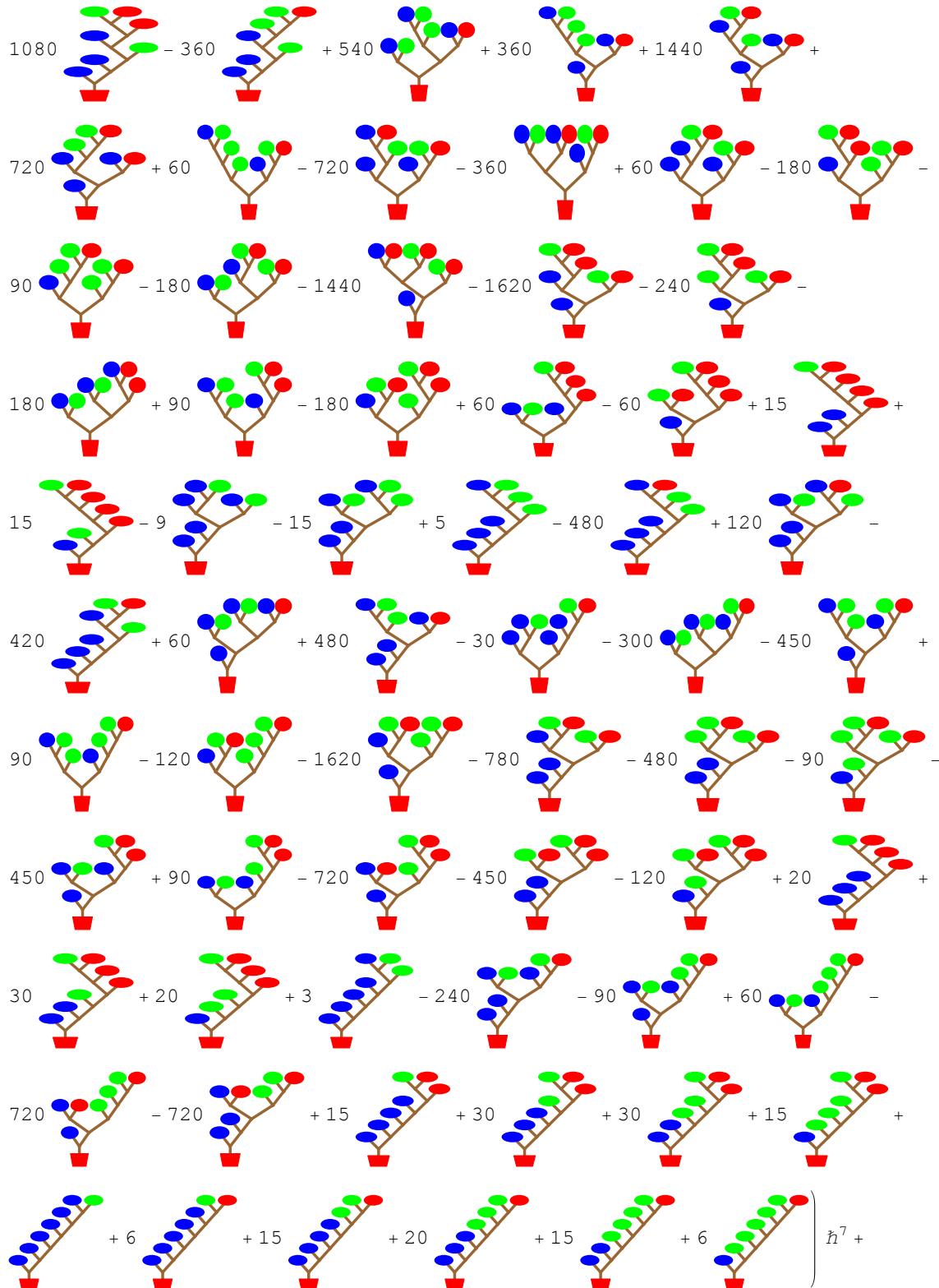
trees = Table[(r /. First[μ0])@k, {k, 8}];
t1 = Series[(List@@trees //.
w_LW :> B@@Reverse[LyndonFactorization[w]] /.
B[s_] :> s /. t_B :> Tree[t]).h^Range[Length[trees]],
{h, 0, Length[trees]}]
] /. {"r" → r, "g" → g, "b" → b};
t1 /. t_Tree :> TreeForm[t,
VertexRenderingFunction → (Switch[#2,
Tree, {
Red,
Polygon[
{{{-0.4, 0.4} - #1, {0.4, 0.4} - #1, {0.3, -0.4} - #1, {-0.3, -0.4} - #1}]
},
B, {},
_, {
ReleaseHold[#2 /. {r → Red, g → Green, b → Blue}],
Disk[-#1, 0.4]
}
]
]&),
EdgeRenderingFunction → ({
Brown, Thickness[0.03],
Line[-#]
}&),
PlotRangePadding → 0, ImageSize → 60, AspectRatio → 1
]

```

$$\begin{aligned}
& h^2 + \frac{1}{2} \left(\text{Diagram 1} + 2 \text{Diagram 2} \right) h^3 + \\
& \frac{1}{12} \left(2 \text{Diagram 3} + 3 \text{Diagram 4} + 6 \text{Diagram 5} + 2 \text{Diagram 6} + 6 \text{Diagram 7} + 6 \text{Diagram 8} \right) h^4 +
\end{aligned}$$

$$\begin{aligned}
 & \frac{1}{24} \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \\ \text{Diagram 5} \\ \text{Diagram 6} \\ \text{Diagram 7} \\ \text{Diagram 8} \\ \text{Diagram 9} \\ \text{Diagram 10} \\ \text{Diagram 11} \\ \text{Diagram 12} \\ \text{Diagram 13} \\ \text{Diagram 14} \\ \text{Diagram 15} \\ \text{Diagram 16} \end{array} \right) - 2 + 2 - 48 - 24 - \\
 & 24 \left(\begin{array}{c} \text{Diagram 17} \\ \text{Diagram 18} \\ \text{Diagram 19} \\ \text{Diagram 20} \\ \text{Diagram 21} \\ \text{Diagram 22} \\ \text{Diagram 23} \\ \text{Diagram 24} \\ \text{Diagram 25} \\ \text{Diagram 26} \\ \text{Diagram 27} \\ \text{Diagram 28} \\ \text{Diagram 29} \\ \text{Diagram 30} \\ \text{Diagram 31} \\ \text{Diagram 32} \end{array} \right) - 12 + 4 + 2 + 12 + \\
 & 6 \left(\begin{array}{c} \text{Diagram 33} \\ \text{Diagram 34} \\ \text{Diagram 35} \\ \text{Diagram 36} \\ \text{Diagram 37} \\ \text{Diagram 38} \\ \text{Diagram 39} \\ \text{Diagram 40} \\ \text{Diagram 41} \\ \text{Diagram 42} \\ \text{Diagram 43} \\ \text{Diagram 44} \\ \text{Diagram 45} \\ \text{Diagram 46} \\ \text{Diagram 47} \\ \text{Diagram 48} \end{array} \right) + 6 + 4 + 4 + 6 + 4 + 4 \Bigg) \hbar^5 + \\
 & \frac{1}{720} \left(\begin{array}{c} \text{Diagram 49} \\ \text{Diagram 50} \\ \text{Diagram 51} \\ \text{Diagram 52} \\ \text{Diagram 53} \\ \text{Diagram 54} \\ \text{Diagram 55} \\ \text{Diagram 56} \\ \text{Diagram 57} \\ \text{Diagram 58} \\ \text{Diagram 59} \\ \text{Diagram 60} \\ \text{Diagram 61} \\ \text{Diagram 62} \\ \text{Diagram 63} \\ \text{Diagram 64} \end{array} \right) - 60 - 720 + 15 - 720 - \\
 & 1080 \left(\begin{array}{c} \text{Diagram 65} \\ \text{Diagram 66} \\ \text{Diagram 67} \\ \text{Diagram 68} \\ \text{Diagram 69} \\ \text{Diagram 70} \\ \text{Diagram 71} \\ \text{Diagram 72} \\ \text{Diagram 73} \\ \text{Diagram 74} \\ \text{Diagram 75} \\ \text{Diagram 76} \\ \text{Diagram 77} \\ \text{Diagram 78} \\ \text{Diagram 79} \\ \text{Diagram 80} \end{array} \right) - 1080 - 360 - 720 + 360 - \\
 & 720 \left(\begin{array}{c} \text{Diagram 81} \\ \text{Diagram 82} \\ \text{Diagram 83} \\ \text{Diagram 84} \\ \text{Diagram 85} \\ \text{Diagram 86} \\ \text{Diagram 87} \\ \text{Diagram 88} \\ \text{Diagram 89} \\ \text{Diagram 90} \\ \text{Diagram 91} \\ \text{Diagram 92} \\ \text{Diagram 93} \\ \text{Diagram 94} \\ \text{Diagram 95} \\ \text{Diagram 96} \end{array} \right) - 2880 - 1080 - 360 - 360 + \\
 & 30 \left(\begin{array}{c} \text{Diagram 97} \\ \text{Diagram 98} \\ \text{Diagram 99} \\ \text{Diagram 100} \\ \text{Diagram 101} \\ \text{Diagram 102} \\ \text{Diagram 103} \\ \text{Diagram 104} \\ \text{Diagram 105} \\ \text{Diagram 106} \\ \text{Diagram 107} \\ \text{Diagram 108} \\ \text{Diagram 109} \\ \text{Diagram 110} \\ \text{Diagram 111} \\ \text{Diagram 112} \end{array} \right) - 30 - 30 + 20 - 720 + 360 - \\
 & 720 \left(\begin{array}{c} \text{Diagram 113} \\ \text{Diagram 114} \\ \text{Diagram 115} \\ \text{Diagram 116} \\ \text{Diagram 117} \\ \text{Diagram 118} \\ \text{Diagram 119} \\ \text{Diagram 120} \\ \text{Diagram 121} \\ \text{Diagram 122} \\ \text{Diagram 123} \\ \text{Diagram 124} \\ \text{Diagram 125} \\ \text{Diagram 126} \\ \text{Diagram 127} \\ \text{Diagram 128} \end{array} \right) + 360 + 720 + 180 - 360 - \\
 & 900 \left(\begin{array}{c} \text{Diagram 129} \\ \text{Diagram 130} \\ \text{Diagram 131} \\ \text{Diagram 132} \\ \text{Diagram 133} \\ \text{Diagram 134} \\ \text{Diagram 135} \\ \text{Diagram 136} \\ \text{Diagram 137} \\ \text{Diagram 138} \\ \text{Diagram 139} \\ \text{Diagram 140} \\ \text{Diagram 141} \\ \text{Diagram 142} \\ \text{Diagram 143} \\ \text{Diagram 144} \end{array} \right) - 240 + 180 - 180 + 60 + 60 + \\
 & 15 \left(\begin{array}{c} \text{Diagram 145} \\ \text{Diagram 146} \\ \text{Diagram 147} \\ \text{Diagram 148} \\ \text{Diagram 149} \\ \text{Diagram 150} \\ \text{Diagram 151} \\ \text{Diagram 152} \\ \text{Diagram 153} \\ \text{Diagram 154} \\ \text{Diagram 155} \\ \text{Diagram 156} \\ \text{Diagram 157} \\ \text{Diagram 158} \\ \text{Diagram 159} \\ \text{Diagram 160} \end{array} \right) - 180 + 180 - 1440 + 60 + 60 + 90 + \\
 & 60 \left(\begin{array}{c} \text{Diagram 161} \\ \text{Diagram 162} \\ \text{Diagram 163} \\ \text{Diagram 164} \\ \text{Diagram 165} \\ \text{Diagram 166} \\ \text{Diagram 167} \\ \text{Diagram 168} \\ \text{Diagram 169} \\ \text{Diagram 170} \\ \text{Diagram 171} \\ \text{Diagram 172} \\ \text{Diagram 173} \\ \text{Diagram 174} \\ \text{Diagram 175} \\ \text{Diagram 176} \end{array} \right) + 6 + 30 + 60 + 60 + 30 + 30 \Bigg) \hbar^6 +
 \end{aligned}$$





$$\begin{aligned}
 & \left(\begin{array}{c} \text{Diagram 1} \\ \text{Diagram 2} \\ \text{Diagram 3} \\ \text{Diagram 4} \\ \text{Diagram 5} \\ \text{Diagram 6} \\ \text{Diagram 7} \\ \text{Diagram 8} \end{array} \right) - \frac{5040}{5} + \frac{12}{12} - \frac{24}{24} - \frac{3}{3} + \frac{144}{144} - \frac{3}{3} - \\
 & \left(\begin{array}{c} \text{Diagram 9} \\ \text{Diagram 10} \\ \text{Diagram 11} \\ \text{Diagram 12} \\ \text{Diagram 13} \\ \text{Diagram 14} \\ \text{Diagram 15} \\ \text{Diagram 16} \end{array} \right) + \frac{3}{4} - \frac{4}{4} + \frac{2}{6} + \frac{7}{4} + \frac{4}{4} + \frac{6}{6} - \\
 & \left(\begin{array}{c} \text{Diagram 17} \\ \text{Diagram 18} \\ \text{Diagram 19} \\ \text{Diagram 20} \\ \text{Diagram 21} \\ \text{Diagram 22} \\ \text{Diagram 23} \\ \text{Diagram 24} \end{array} \right) + \frac{12}{12} - \frac{12}{12} - \frac{8}{8} - \frac{6}{6} + \frac{2}{2} - \frac{24}{24} + \frac{12}{12} + \\
 & \left(\begin{array}{c} \text{Diagram 25} \\ \text{Diagram 26} \\ \text{Diagram 27} \\ \text{Diagram 28} \\ \text{Diagram 29} \\ \text{Diagram 30} \\ \text{Diagram 31} \\ \text{Diagram 32} \end{array} \right) - \frac{72}{72} + \frac{4}{4} - \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{8}{8} + \\
 & \left(\begin{array}{c} \text{Diagram 33} \\ \text{Diagram 34} \\ \text{Diagram 35} \\ \text{Diagram 36} \\ \text{Diagram 37} \\ \text{Diagram 38} \\ \text{Diagram 39} \\ \text{Diagram 40} \end{array} \right) - \frac{24}{24} - \frac{6}{6} - \frac{12}{12} + \frac{1440}{1440} - \frac{4}{4} - \frac{8}{8} + \frac{4}{4} - \\
 & \left(\begin{array}{c} \text{Diagram 41} \\ \text{Diagram 42} \\ \text{Diagram 43} \\ \text{Diagram 44} \\ \text{Diagram 45} \\ \text{Diagram 46} \\ \text{Diagram 47} \\ \text{Diagram 48} \end{array} \right) - \frac{8}{8} - \frac{12}{12} - \frac{6}{6} + \frac{12}{12} - \frac{6}{6} - \frac{6}{6} + \frac{3}{3} + \\
 & \left(\begin{array}{c} \text{Diagram 49} \\ \text{Diagram 50} \\ \text{Diagram 51} \\ \text{Diagram 52} \\ \text{Diagram 53} \\ \text{Diagram 54} \\ \text{Diagram 55} \\ \text{Diagram 56} \end{array} \right) + \frac{4}{4} - \frac{12}{12} + \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{8}{8} + \frac{4}{4} - \\
 & \left(\begin{array}{c} \text{Diagram 57} \\ \text{Diagram 58} \\ \text{Diagram 59} \\ \text{Diagram 60} \\ \text{Diagram 61} \\ \text{Diagram 62} \\ \text{Diagram 63} \\ \text{Diagram 64} \end{array} \right) - \frac{4}{4} - \frac{4}{4} + \frac{3}{3} + \frac{2}{2} - \frac{4}{4} + \frac{4}{4} + \frac{4}{4} - \frac{4}{4}
 \end{aligned}$$

$$\begin{array}{ccccccc}
 \text{Diagram 1} & + & \text{Diagram 2} & - & \text{Diagram 3} & + & \text{Diagram 4} \\
 \frac{2}{\text{Diagram 1}} & & \frac{8}{\text{Diagram 2}} & & \frac{4}{\text{Diagram 3}} & & \frac{24}{\text{Diagram 4}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 5} & + & \text{Diagram 6} & - & \text{Diagram 7} & + & \text{Diagram 8} \\
 \frac{6}{\text{Diagram 5}} & & \frac{4}{\text{Diagram 6}} & & \frac{6}{\text{Diagram 7}} & & \frac{12}{\text{Diagram 8}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 9} & + & \text{Diagram 10} & - & \text{Diagram 11} & + & \text{Diagram 12} \\
 \frac{6}{\text{Diagram 9}} & & \frac{24}{\text{Diagram 10}} & & \frac{6}{\text{Diagram 11}} & & \frac{2}{\text{Diagram 12}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 13} & + & \text{Diagram 14} & + & \text{Diagram 15} & + & \text{Diagram 16} \\
 \frac{2}{\text{Diagram 13}} & & \frac{3}{\text{Diagram 14}} & & \frac{2}{\text{Diagram 15}} & & \frac{4}{\text{Diagram 16}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 17} & - & \text{Diagram 18} & - & \text{Diagram 19} & + & \text{Diagram 20} \\
 \frac{6}{\text{Diagram 17}} & & \frac{3}{\text{Diagram 18}} & & \frac{8}{\text{Diagram 19}} & & \frac{4}{\text{Diagram 20}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 21} & - & \text{Diagram 22} & + & \text{Diagram 23} & + & \text{Diagram 24} \\
 \frac{5}{\text{Diagram 21}} & & \frac{6}{\text{Diagram 22}} & & \frac{3}{\text{Diagram 23}} & & \frac{2}{\text{Diagram 24}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 25} & - & \text{Diagram 26} & + & \text{Diagram 27} & - & \text{Diagram 28} \\
 \frac{5}{\text{Diagram 25}} & & \frac{2}{\text{Diagram 26}} & & \frac{4}{\text{Diagram 27}} & & \frac{12}{\text{Diagram 28}}
 \end{array}$$

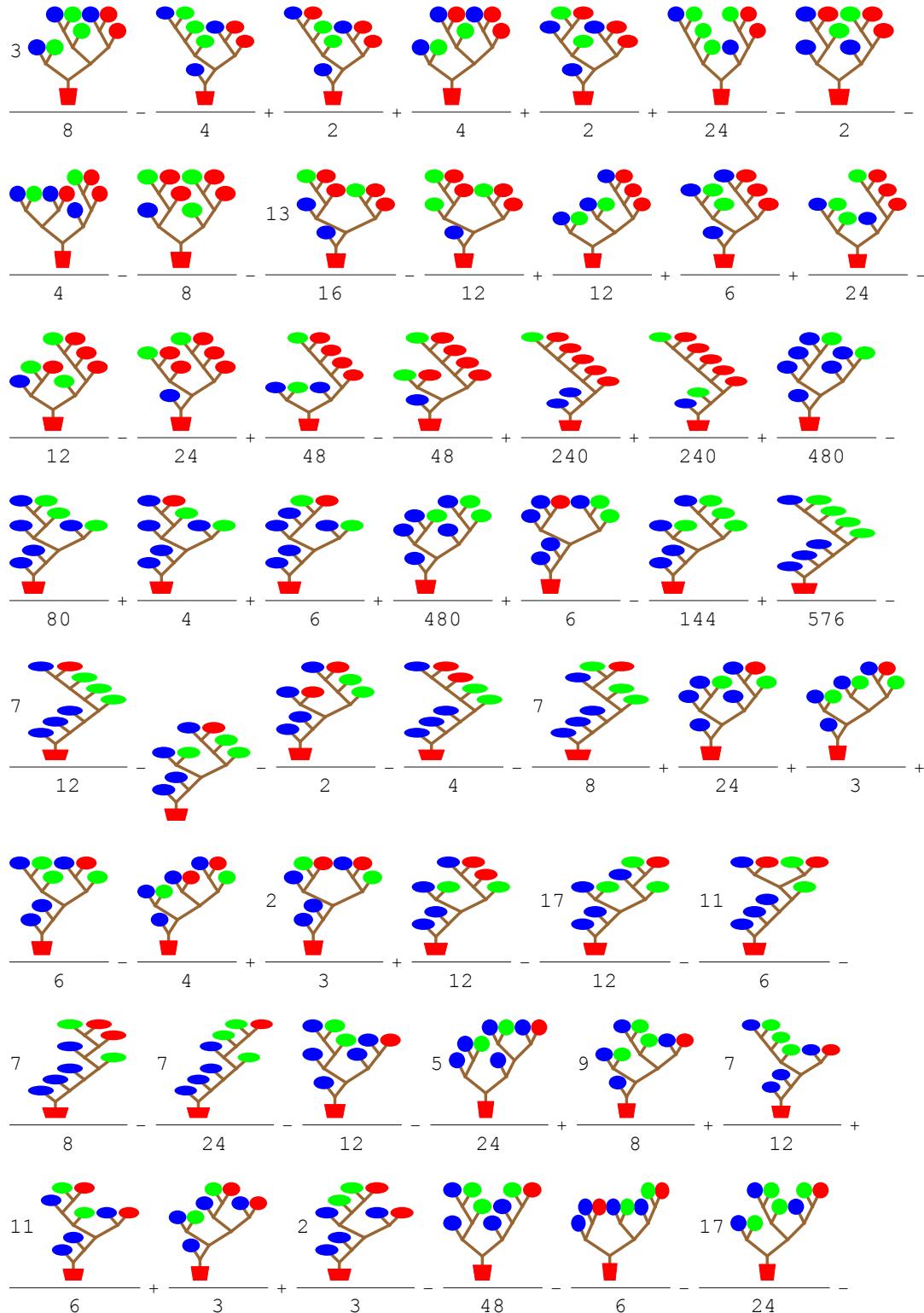
$$\begin{array}{ccccccc}
 \text{Diagram 29} & - & \text{Diagram 30} & + & \text{Diagram 31} & - & \text{Diagram 32} \\
 \frac{3}{\text{Diagram 29}} & & \frac{4}{\text{Diagram 30}} & & \frac{24}{\text{Diagram 31}} & & \frac{4}{\text{Diagram 32}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 33} & + & \text{Diagram 34} & + & \text{Diagram 35} & + & \text{Diagram 36} \\
 \frac{720}{\text{Diagram 33}} & & \frac{240}{\text{Diagram 34}} & & \frac{4}{\text{Diagram 35}} & & \frac{180}{\text{Diagram 36}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 37} & + & \text{Diagram 38} & - & \text{Diagram 39} & + & \text{Diagram 40} \\
 \frac{720}{\text{Diagram 37}} & & \frac{12}{\text{Diagram 38}} & & \frac{72}{\text{Diagram 39}} & & \frac{72}{\text{Diagram 40}}
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 1} & + & \text{Diagram 2} & - & \text{Diagram 3} & + & \text{Diagram 4} \\
 \frac{5}{24} & & \frac{5}{24} & & \frac{2}{3} & & \frac{4}{12} \\
 \\
 \text{Diagram 5} & - & \text{Diagram 6} & + & \text{Diagram 7} & - & \text{Diagram 8} \\
 \frac{12}{-} & & \frac{96}{+} & & \frac{2}{-} & & \frac{6}{+} \\
 \\
 \text{Diagram 9} & + & \text{Diagram 10} & - & \text{Diagram 11} & + & \text{Diagram 12} \\
 \frac{288}{+} & & \frac{720}{-} & & \frac{12}{-} & & \frac{17}{+} \\
 \\
 \text{Diagram 13} & + & \text{Diagram 14} & - & \text{Diagram 15} & + & \text{Diagram 16} \\
 \frac{7}{6} & & \frac{3}{8} & & \frac{3}{4} & & \frac{9}{8} \\
 \\
 \text{Diagram 17} & + & \text{Diagram 18} & - & \text{Diagram 19} & + & \text{Diagram 20} \\
 \frac{4}{+} & & \frac{43}{8} & & \frac{2}{-} & & \frac{13}{2} \\
 \\
 \text{Diagram 21} & - & \text{Diagram 22} & + & \text{Diagram 23} & + & \text{Diagram 24} \\
 \frac{3}{4} & & \frac{3}{4} & & \frac{7}{24} & & \frac{5}{12} \\
 \\
 \text{Diagram 25} & + & \text{Diagram 26} & - & \text{Diagram 27} & + & \text{Diagram 28} \\
 \frac{4}{+} & & \frac{2}{-} & & \frac{3}{4} & & \frac{7}{24} \\
 \\
 \text{Diagram 29} & - & \text{Diagram 30} & + & \text{Diagram 31} & - & \text{Diagram 32} \\
 \frac{8}{-} & & \frac{4}{+} & & \frac{4}{+} & & \frac{3}{-} \\
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 1} & + & \text{Diagram 2} & + & \text{Diagram 3} & - & \text{Diagram 4} \\
 \frac{2}{3} & & \frac{2}{2} & & \frac{11}{2} & & \frac{3}{2} \\
 \\
 \text{Diagram 5} & - & \text{Diagram 6} & - & \text{Diagram 7} & - & \text{Diagram 8} \\
 \frac{5}{2} & & \frac{7}{6} & & \frac{3}{2} & & \frac{24}{24} \\
 \\
 \text{Diagram 9} & + & \text{Diagram 10} & + & \text{Diagram 11} & + & \text{Diagram 12} \\
 \frac{5}{24} & & \frac{5}{12} & & \frac{3}{8} & & \frac{7}{12} \\
 \\
 \text{Diagram 13} & + & \text{Diagram 14} & + & \text{Diagram 15} & + & \text{Diagram 16} \\
 \frac{3}{2} & & \frac{3}{2} & & \frac{6}{6} & & \frac{2}{2} \\
 \\
 \text{Diagram 17} & - & \text{Diagram 18} & + & \text{Diagram 19} & + & \text{Diagram 20} \\
 \frac{48}{48} & & \frac{2}{2} & & \frac{2}{2} & & \frac{12}{12} \\
 \\
 \text{Diagram 21} & + & \text{Diagram 22} & - & \text{Diagram 23} & + & \text{Diagram 24} \\
 \frac{12}{12} & & \frac{4}{4} & & \frac{12}{8} & & \frac{3}{3} \\
 \\
 \text{Diagram 25} & + & \text{Diagram 26} & + & \text{Diagram 27} & - & \text{Diagram 28} \\
 \frac{12}{12} & & \frac{8}{8} & & \frac{4}{4} & & \frac{17}{12} \\
 \\
 \text{Diagram 29} & - & \text{Diagram 30} & - & \text{Diagram 31} & - & \text{Diagram 32} \\
 \frac{9}{9} & & \frac{12}{12} & & \frac{8}{4} & & \frac{3}{3}
 \end{array}$$



$$\begin{array}{ccccccc}
 \text{Diagram 13} & + & \text{Diagram 2} & + & \text{Diagram 5} & - & \text{Diagram 11} \\
 \frac{1}{24} & & \frac{1}{3} & & \frac{1}{24} & & \frac{1}{24} \\
 \\
 \text{Diagram 2} & - & \text{Diagram 4} & + & \text{Diagram 12} & - & \text{Diagram 12} \\
 \frac{1}{2} & & \frac{1}{4} & & \frac{1}{12} & & \frac{1}{12} \\
 & & & & & + & \text{Diagram 2} \\
 & & & & & & \frac{1}{3} \\
 \\
 \text{Diagram 4} & + & \text{Diagram 2} & + & \text{Diagram 8} & - & \text{Diagram 16} \\
 & & & & & & \frac{1}{16} \\
 & & & & & & \frac{1}{60} \\
 & & & & & + & \text{Diagram 2} \\
 & & & & & & \frac{1}{2} \\
 & & & & & & + & \text{Diagram 2} \\
 & & & & & & & - \\
 \\
 \text{Diagram 2} & - & \text{Diagram 2} & + & \text{Diagram 8} & - & \text{Diagram 6} \\
 & & & & & & \frac{1}{6} \\
 & & & & & & \frac{1}{12} \\
 & & & & & & \frac{1}{25} \\
 & & & & & & - & \text{Diagram 7} \\
 & & & & & & & \frac{1}{6} \\
 \\
 \text{Diagram 8} & + & \text{Diagram 8} & + & \text{Diagram 4} & - & \text{Diagram 48} \\
 & & & & & & \frac{1}{48} \\
 & & & & & & \frac{1}{24} \\
 & & & & & & \frac{1}{16} \\
 & & & & & & \frac{1}{25} \\
 & & & & & & - & \text{Diagram 16} \\
 & & & & & & & \frac{1}{24} \\
 & & & & & & & + \\
 \\
 \text{Diagram 16} & - & \text{Diagram 12} & - & \text{Diagram 8} & - & \text{Diagram 16} \\
 & & & & & & \frac{1}{8} \\
 & & & & & & \frac{1}{12} \\
 & & & & & & \frac{1}{13} \\
 & & & & & & \frac{1}{8} \\
 & & & & & & \frac{1}{16} \\
 & & & & & & \frac{1}{2} \\
 & & & & & & \frac{1}{8} \\
 & & & & & & + & \text{Diagram} \\
 & & & & & & & \frac{1}{16} \\
 \\
 \text{Diagram 12} & - & \text{Diagram 24} & + & \text{Diagram 24} & - & \text{Diagram 3} \\
 & & & & & & \frac{1}{3} \\
 & & & & & & \frac{1}{24} \\
 & & & & & & \frac{1}{24} \\
 & & & & & & \frac{1}{18} \\
 & & & & & & \frac{1}{720} \\
 & & & & & & \frac{1}{4} \\
 & & & & & & + & \text{Diagram} \\
 & & & & & & & \frac{1}{24} \\
 & & & & & & & - \\
 \\
 \text{Diagram 96} & + & \text{Diagram 144} & - & \text{Diagram 360} & - & \text{Diagram 160} \\
 & & & & & & \frac{1}{160} \\
 & & & & & & \frac{1}{720} \\
 & & & & & & \frac{1}{4} \\
 & & & & & & + & \text{Diagram} \\
 & & & & & & & \frac{1}{24} \\
 & & & & & & & - \\
 \end{array}$$

$$\begin{array}{ccccccc}
 \text{Diagram 1} & + & \text{Diagram 2} & + & \text{Diagram 3} & - & \text{Diagram 4} \\
 \frac{4}{4} & & \frac{12}{12} & & \frac{12}{48} & & \frac{4}{4} \\
 \\
 \text{Diagram 5} & - & \text{Diagram 6} & + & \text{Diagram 7} & + & \text{Diagram 8} \\
 \frac{5}{24} & & \frac{5}{16} & & \frac{3}{24} & & \frac{17}{12} \\
 \\
 \text{Diagram 9} & - & \text{Diagram 10} & - & \text{Diagram 11} & - & \text{Diagram 12} \\
 \frac{2}{2} & & \frac{12}{12} & & \frac{6}{6} & & \frac{9}{48} \\
 \\
 \text{Diagram 13} & - & \text{Diagram 14} & - & \text{Diagram 15} & + & \text{Diagram 16} \\
 \frac{11}{48} & & \frac{7}{30} & & \frac{12}{16} & & \frac{2}{24} \\
 \\
 \text{Diagram 17} & - & \text{Diagram 18} & - & \text{Diagram 19} & + & \text{Diagram 20} \\
 \frac{13}{24} & & \frac{3}{3} & & \frac{16}{144} & + & \frac{72}{72} \\
 \\
 \text{Diagram 21} & - & \text{Diagram 22} & - & \text{Diagram 23} & + & \text{Diagram 24} \\
 \frac{1440}{1440} & & \frac{16}{16} & & \frac{6}{24} & + & \frac{3}{48} \\
 \\
 \text{Diagram 25} & - & \text{Diagram 26} & - & \text{Diagram 27} & + & \text{Diagram 28} \\
 \frac{2}{3} & & \frac{240}{240} & & \frac{96}{96} & + & \frac{72}{72} \\
 \\
 \text{Diagram 29} & - & \text{Diagram 30} & - & \text{Diagram 31} & + & \text{Diagram 32} \\
 \frac{720}{720} & & \frac{240}{240} & & \frac{144}{144} & + & \frac{144}{240} \\
 \\
 \text{Diagram 33} & - & \text{Diagram 34} & - & \text{Diagram 35} & + & \text{Diagram 36} \\
 \frac{720}{720} & & \frac{240}{240} & & \frac{144}{144} & + & \frac{240}{720}
 \end{array}$$

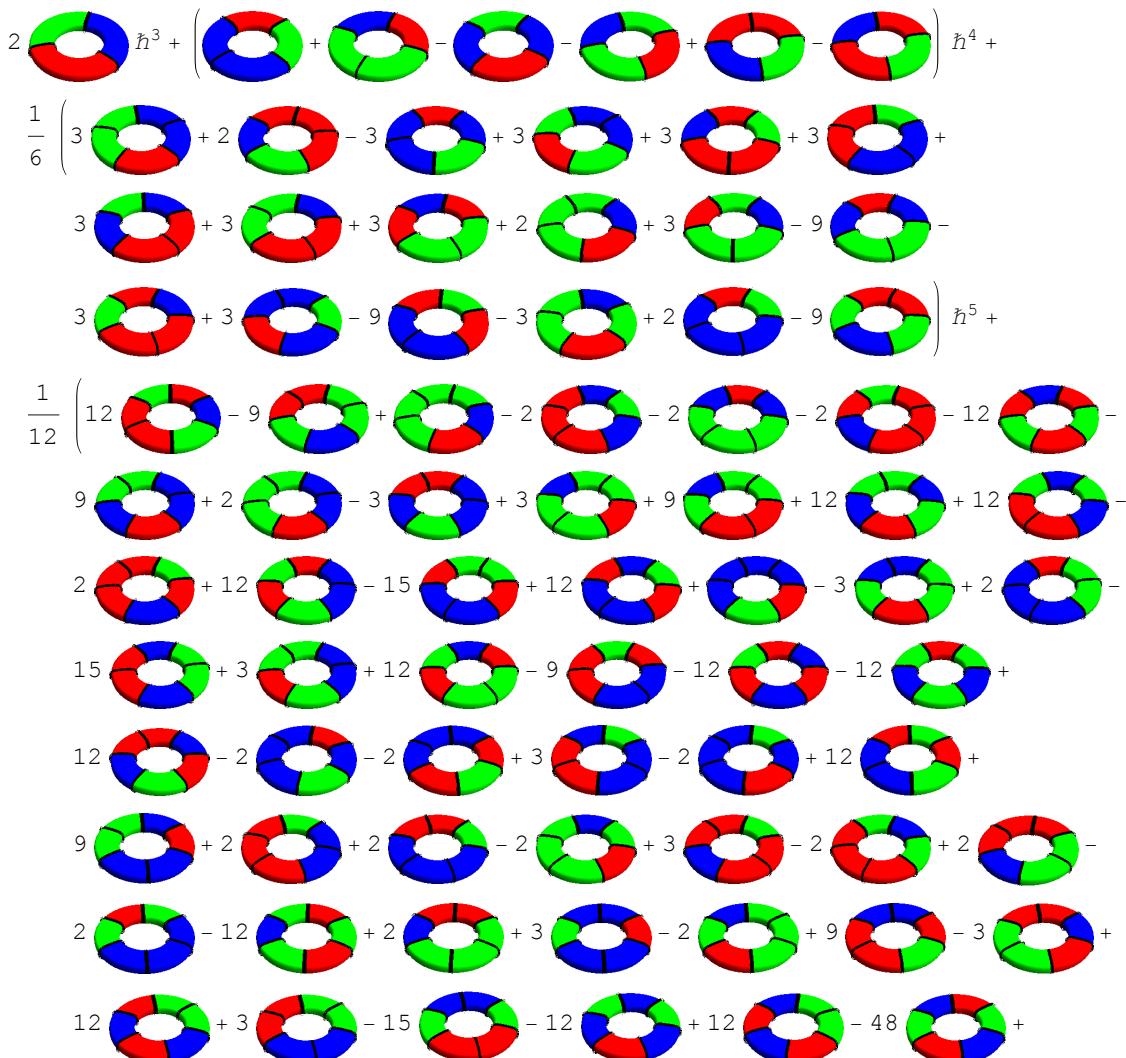
$\left. \right\} \hbar^8 + O[\hbar]^9$

Wheels

```

n = 7;
wheels = Table[Last[μ0]@k, {k, n}];
SetOptions[Rasterize, {RasterSize → 256, ImageSize → 256}];
Collect[
  Expand[(Plus @@ wheels)] /.
    CW[s_String] → ℏStringLength[s] Show[ImageCrop[PieChart3D[
      Table[1, {StringLength[s]}],
      ChartStyle → (Characters[s] /. {"r" → Red, "g" → Green, "b" → Blue}),
      SectorOrigin → {{RandomReal[{0, 2 π}], "Counterclockwise"}, 1},
      ChartBaseStyle → EdgeForm[{Thickness[0.03], Black}],
      ChartElementFunction → "ProfileSector3D",
      ImagePadding → 0, ImageMargins → 0, PlotRangePadding → 0
    ]], ImageSize → 52],
  ℏ, Factor] + O[ℏ]^{n+1}

```



$$\begin{aligned}
 & 12 \left(+ 12 \left(- 2 \left(+ 1 \left(- 12 \left(\dots \right) \right) \right) \right) \right) \hbar^6 + \\
 & \frac{1}{360} \left(- 180 \left(+ 180 \left(+ 360 \left(+ 30 \left(+ 15 \left(- 30 \left(\dots \right) \right) \right) \right) \right) \right) \right. \\
 & 180 \left(- 540 \left(+ 180 \left(- 720 \left(- 360 \left(+ 20 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 30 \left(+ 135 \left(- 180 \left(+ 30 \left(+ 180 \left(- 225 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 360 \left(- 180 \left(- 45 \left(+ 180 \left(+ 30 \left(- 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 180 \left(+ 20 \left(- 360 \left(+ 15 \left(- 90 \left(- 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 180 \left(- 360 \left(- 180 \left(+ 30 \left(+ 360 \left(- 90 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 180 \left(- 180 \left(+ 180 \left(+ 180 \left(+ 180 \left(+ 1080 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 90 \left(- 180 \left(+ 225 \left(+ 180 \left(- 90 \left(- 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 180 \left(+ 360 \left(- 180 \left(+ 180 \left(+ 360 \left(+ 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 30 \left(- 30 \left(+ 180 \left(+ 30 \left(- 180 \left(+ 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 360 \left(- 30 \left(+ 135 \left(- 360 \left(+ 30 \left(+ 360 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 180 \left(+ 360 \left(- 180 \left(- 180 \left(+ 360 \left(+ 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 15 \left(- 720 \left(+ 180 \left(- 30 \left(+ 30 \left(+ 360 \left(+ 180 \left(\dots \right) \right) \right) \right) \right) \right) \right. \\
 & 180 \left(- 180 \left(+ 1080 \left(+ 360 \left(- 225 \left(+ 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 15 \left(+ 360 \left(- 30 \left(+ 15 \left(- 540 \left(- 180 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 15 \left(+ 30 \left(- 1080 \left(+ 360 \left(+ 15 \left(- 45 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 90 \left(+ 225 \left(- 360 \left(- 1080 \left(+ 180 \left(+ 30 \left(\dots \right) \right) \right) \right) \right) \right. \\
 & 180 \left(- 360 \left(+ 180 \left(- 90 \left(- 360 \left(+ 360 \left(\dots \right) \right) \right) \right) \right) \right.
 \end{aligned}$$

