Cheat Sheet J

http://drorbn.net/AcademicPensieve/2013-03/
initiated 18/3/13; modified 4/4/13, 9:26am; continued 2013-04

With alphabet T and with $u,v,w\in T$, $\alpha,\beta,\gamma\in FL(T)$, $D\in\mathfrak{tder}(T)$, $g,h\in\exp(\mathfrak{tder}(T))=\mathrm{TAut}(T)$. Checkmarks (✓) as in `CheatSheetJ-Verification.nb`.

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1. The definition of J: $$J_u(\gamma):=\int_0^1 ds\,\mathrm{div}_u(\gamma /\!\!/ RC_u^{s\gamma}) /\!\!/ C_u^{-s\gamma}$$
2. ✓ The t equation (desired): $$J_w(\gamma /\!\!/ tm_w^{uv}) /\!\!/ RC_w^{\gamma /\!\!/ tm_w^{uv}} = J_u(\gamma) /\!\!/ tm_w^{uv} /\!\!/ RC_w^{\gamma /\!\!/ tm_w^{uv}} + J_v(\gamma /\!\!/ RC_u^\gamma) /\!\!/ RC_v^{\gamma /\!\!/ RC_u^\gamma} /\!\!/ tm_w^{uv}$$
3. ✓ The h equation (desired): $$J_u(\mathrm{bch}(\alpha,\beta)) = J_u(\alpha) + J_u(\beta /\!\!/ RC_u^\alpha) /\!\!/ C_u^{-\alpha}$$

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4. ✓ The meaning(s) of RC: $$CC_u^\gamma /\!\!/ RC_u^{-\gamma} = Id, \qquad CC_u^{\gamma /\!\!/ RC_u^\gamma} = RC_u^\gamma$$
5. RC equation t: $$tm_w^{uv} /\!\!/ RC_w^{\gamma /\!\!/ tm_w^{uv}} = RC_u^\gamma /\!\!/ RC_v^{\gamma /\!\!/ RC_u^\gamma} /\!\!/ tm_w^{uv}$$
6. RC equation h: $$RC_u^{\mathrm{bch}(\alpha,\beta)} = RC_u^\alpha /\!\!/ RC_u^{\beta /\!\!/ RC_u^\alpha}$$
7. RCC equation div: $$\mathrm{div}_u(\alpha /\!\!/ RC_u^\gamma) /\!\!/ C_u^\gamma = ?$$
8. CRC equation div: $$\mathrm{div}_u(\alpha /\!\!/ C_u^\gamma) /\!\!/ RC_u^\gamma = ?$$

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9. div property t: $$\mathrm{div}_w(\gamma /\!\!/ tm_w^{uv}) = (\mathrm{div}_u(\gamma)+\mathrm{div}_v(\gamma)) /\!\!/ tm_w^{uv}$$
10. ✓ div property h — the "cocycle condition": with $\mathrm{ad}_u\{\gamma\}:=\mathrm{der}(u\to[\gamma,u])$, $$(\mathrm{div}_u\,\alpha) /\!\!/ \mathrm{ad}_u\{\beta\} - (\mathrm{div}_u\,\beta) /\!\!/ \mathrm{ad}_u\{\alpha\} = \mathrm{div}_u\left([\alpha,\beta] + \alpha /\!\!/ \mathrm{ad}_u\{\beta\} - \beta /\!\!/ \mathrm{ad}_u\{\alpha\}\right)$$
11. div of bch: $$\mathrm{div}_u(\mathrm{bch}(\alpha,\beta)) = ?$$

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12. The definition of JA: $$JA_u(\gamma) := J_u(\gamma) /\!\!/ RC_u^\gamma$$
13. The ODE for JA: with $\gamma_s=\gamma /\!\!/ RC_u^{s\gamma}$, $$JA(0)=0, \quad \frac{dJA(s)}{ds} = JA(s) /\!\!/ \mathrm{ad}_u\{\gamma_s\} + \mathrm{div}_u\,\gamma_s, \quad JA(1)=JA_u(\gamma)$$

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14. The relation with $\mathfrak{tder}$: $$e^{\mathrm{ad}_u\{\gamma\}} = C_u^{?} \text{ and } C_u^\gamma = e^{\mathrm{ad}_u\{?\}}$$
15. The definition of j (following A-T): $$j(e^D) = \int_0^1 ds\, e^{sD}(\mathrm{div}\,D) = \frac{e^D-1}{D}(\mathrm{div}\,D)$$
16. j's cocycle property: $$j(gh) = j(g) + g\cdot j(h)$$

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17. The differential of exp: $$\delta e^\gamma = e^\gamma\cdot\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) = \left(\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right)(\delta\gamma)\cdot e^\gamma$$
18. ✓ The differential of $\gamma=\mathrm{bch}(\alpha,\beta)$: $$\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) = \left(e^{-\mathrm{ad}\,\beta}\frac{1-e^{-\mathrm{ad}\,\alpha}}{\mathrm{ad}\,\alpha}\right)(\delta\alpha) + \left(\frac{1-e^{-\mathrm{ad}\,\beta}}{\mathrm{ad}\,\beta}\right)(\delta\beta)$$
19. ✓ The differential of C: $$\delta C_u^\gamma = \mathrm{ad}_u\left\{\left(\frac{e^{\mathrm{ad}\,\gamma}-1}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) /\!\!/ RC_u^{-\gamma}\right\} /\!\!/ C_u^\gamma$$
20. ✓ The differential of RC: $$\delta RC_u^\gamma = RC_u^\gamma /\!\!/ \mathrm{ad}_u\left\{\left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right)(\delta\gamma) /\!\!/ RC_u^\gamma\right\}$$

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21. ✓ The differential of J: $$\delta J_u(\gamma) = \delta\gamma /\!\!/ \left(\frac{1-e^{-\mathrm{ad}\,\gamma}}{\mathrm{ad}\,\gamma}\right) /\!\!/ RC_u^\gamma /\!\!/ \mathrm{div}_u /\!\!/ C_u^{-\gamma}$$

Recycling.

✓ The differential of J:

$$\delta J_u(\gamma) = \int_0^1 ds\ \mathrm{div}_u\left(\delta\gamma /\!\!/ e^{-\operatorname{ad} s\gamma} /\!\!/ RC_u^{s\gamma}\right) /\!\!/ C_u^{-s\gamma}$$
$$+ \int_0^1 ds\ \mathrm{div}_u\left(\left(\frac{1-e^{-\operatorname{ad} s\gamma}}{\operatorname{ad}\gamma}\right)(\delta\gamma) /\!\!/ RC_u^{s\gamma} /\!\!/ \operatorname{ad}_u\{\gamma /\!\!/ RC_u^{s\gamma}\}\right) /\!\!/ C_u^{-s\gamma}$$
$$- \int_0^1 ds\ \mathrm{div}_u\left(\left(\frac{1-e^{-\operatorname{ad} s\gamma}}{\operatorname{ad}\gamma}\right)(\delta\gamma) /\!\!/ RC_u^{s\gamma}\right) /\!\!/ \operatorname{ad}_u\{\gamma /\!\!/ RC_u^{s\gamma}\} /\!\!/ C_u^{-s\gamma}$$