

Pensieve header: To be run after executing JExperiments-Program.nb.

### Testing tm

Testing\_tm

```
{ {u = <"u">, v = <"v">, w = <"w">} ;
t1 = M[{
  x → MakeLieSeries[u + v + w],
  y → MakeLieSeries[b[u, v] + b[v, w]]
}, MakeCWSeries[CW["uvw"]]
],
t1 // tm[u, v, u],
t2 = t1 // tm[u, v, u] // tm[u, w, u],
t3 = t1 // tm[v, w, v] // tm[u, v, u],
t2 ≡ t3
} // ColumnForm
```

Testing\_tm

```
M[{x → LS[u + v + w, 0, 0], y → LS[0,  $\overline{uv} + \overline{vw}$ , 0]}, CWS[0, 0, CW["uvw"]]]
M[{x → LS[2 u + w, 0, 0], y → LS[0,  $\overline{uw}$ , 0]}, CWS[0, 0, CW["uuw"]]]
M[{x → LS[3 u, 0, 0], y → LS[0, 0, 0]}, CWS[0, 0, CW["uuu"]]]
M[{x → LS[3 u, 0, 0], y → LS[0, 0, 0]}, CWS[0, 0, CW["uuu"]]]
True
```

### Testing hm

Testing\_hm

```
{ t1 = R+[u, x] ∪ R+[v, y] ∪ R+[w, z],
t1 // hm[x, y, x],
t2 = t1 // hm[x, y, x] // hm[x, z, x],
t1 // hm[y, z, y],
t3 = t1 // hm[y, z, y] // hm[x, y, x],
t2 ≡ t3
} // ColumnForm
```

Testing\_hm

```
M[{x → LS[u, 0, 0], y → LS[v, 0, 0], z → LS[w, 0, 0], ⟨u⟩ → LS[0, 0, 0], ⟨v⟩ → LS[0, 0, 0], ⟨v
M[ $\left\{x \rightarrow LS\left[u+v, \frac{\overline{uv}}{2}, \frac{1}{12}\overline{u\overline{uv}} + \frac{1}{12}\overline{u\overline{v}v}\right], z \rightarrow LS[w, 0, 0], \langle u \rangle \rightarrow LS[0, 0, 0], \langle v \rangle \rightarrow LS[0, 0, 0]\right.$ 
M[ $\left\{x \rightarrow LS\left[u+v+w, \frac{\overline{uv}}{2} + \frac{\overline{uw}}{2} + \frac{\overline{vw}}{2}, \frac{1}{12}\overline{u\overline{uv}} + \frac{1}{12}\overline{u\overline{uw}} + \frac{1}{3}\overline{u\overline{vw}} + \frac{1}{12}\overline{v\overline{vw}} + \frac{1}{12}\overline{u\overline{v}v} + \frac{1}{6}\overline{u\overline{w}v} + \frac{1}{12}\overline{u}\right]$ 
M[ $\left\{x \rightarrow LS[u, 0, 0], y \rightarrow LS\left[v+w, \frac{\overline{vw}}{2}, \frac{1}{12}\overline{v\overline{vw}} + \frac{1}{12}\overline{v\overline{ww}}\right], \langle u \rangle \rightarrow LS[0, 0, 0], \langle v \rangle \rightarrow LS[0, 0, 0]\right.$ 
M[ $\left\{x \rightarrow LS\left[u+v+w, \frac{\overline{uv}}{2} + \frac{\overline{uw}}{2} + \frac{\overline{vw}}{2}, \frac{1}{12}\overline{u\overline{uv}} + \frac{1}{12}\overline{u\overline{uw}} + \frac{1}{3}\overline{u\overline{vw}} + \frac{1}{12}\overline{v\overline{vw}} + \frac{1}{12}\overline{u\overline{v}v} + \frac{1}{6}\overline{u\overline{w}v} + \frac{1}{12}\overline{u}\right]$ 
True
```

## Testing C and RC

```
$SeriesShowDegree = 6;
{{u = <"u">, w = <"w">},
 λ0 = MakeLieSeries[u + <"uw">],
 t1 = MakeLieSeries[<"uv">],
 t2 = t1 // LieMorphism[u → Ad[-λ0][u]],
 t3 = t2 // RC[u, λ0],
 t1 ≡ t3
} // ColumnForm
{⟨u⟩, ⟨w⟩}
LS[u, w, 0, 0, 0, 0]
LS[0, v, 0, 0, 0, 0]
LS[0, wv, uwv, - $\frac{1}{2}$  uuvwv -  $\frac{1}{2}$  uvuwv +  $\frac{1}{2}$  uuvwuv,  $\frac{1}{6}$  uuuwvw +  $\frac{1}{6}$  uuvuwv +  $\frac{1}{2}$  uvuwvw
LS[0, v, 0, 0, 0, 0]
True
```

## Testing the RC “composition property”

```
$SeriesShowDegree = 4;
{{u = <"u">, v = <"v">, w = <"w">},
 λ = MakeLieSeries[u],
 λx = MakeLieSeries[v],
 λy = MakeLieSeries[w],
 BCH[λx, λy],
 t1 = λ // RC[u, BCH[λx, λy]],
 t2 = λ // RC[u, λx] // RC[u, λy] // RC[u, λx],
 t1 ≡ t2
} // ColumnForm
{⟨u⟩, ⟨v⟩, ⟨w⟩}
LS[u, 0, 0, 0]
LS[v, 0, 0, 0]
LS[w, 0, 0, 0]
LS[v + w,  $\frac{vw}{2}$ ,  $\frac{1}{12}$  vvw +  $\frac{1}{12}$  vww,  $\frac{1}{24}$  vvwvw]
LS[u, -uv - w,  $\frac{1}{2}$  uvv + uwv +  $\frac{1}{2}$  uww, - $\frac{1}{6}$  uvvv -  $\frac{1}{2}$  uwvv -  $\frac{1}{2}$  uwwv -  $\frac{1}{6}$  uwww
LS[u, -uv - w,  $\frac{1}{2}$  uvv + uwv +  $\frac{1}{2}$  uww, - $\frac{1}{6}$  uvvv -  $\frac{1}{2}$  uwvv -  $\frac{1}{2}$  uwwv -  $\frac{1}{6}$  uwww
True
```

```

$SeriesShowDegree = 4;
$SeriesCompareDegree = 6;
{{u = <"u">, v = <"v">, w = <"w">},
 λ = MakeLieSeries[u],
 λx = MakeLieSeries[v + b[u, v] + b[u, b[u, v]]],
 λy = MakeLieSeries[w + b[u, b[v, w]]],
 BCH[λx, λy],
 t1 = λ // RC[u, BCH[λx, λy]],
 t2 = λ // RC[u, λx] // RC[u, λy] // RC[u, λx]],
 t1 ≡ t2
} // ColumnForm

{⟨u⟩, ⟨v⟩, ⟨w⟩}
LS[u, 0, 0, 0]
LS[v,  $\overline{uv}$ ,  $\overline{u\overline{uv}}$ , 0]
LS[w, 0,  $\overline{uvw}$ , 0]
LS[v + w,  $\overline{uv} + \frac{\overline{vw}}{2}$ ,  $\overline{u\overline{uv}}$  +  $\frac{3}{2}\overline{u\overline{vw}}$  +  $\frac{1}{12}\overline{v\overline{vw}}$  +  $\frac{1}{2}\overline{u\overline{vw}}$  +  $\frac{1}{12}\overline{vw\overline{w}}$ ,  $\frac{1}{2}\overline{u\overline{u\overline{vw}}}$  +  $\frac{1}{12}\overline{u\overline{v\overline{vw}}}$  +  $\frac{1}{2}\overline{u\overline{u\overline{wv}}}$  +
LS[u, - $\overline{uv} - \overline{uw}$ , - $\overline{u\overline{uv}}$  +  $\frac{1}{2}\overline{u\overline{vv}}$  +  $\overline{u\overline{wv}}$  +  $\frac{1}{2}\overline{u\overline{ww}}$ , - $\overline{u\overline{u\overline{uv}}}$  -  $\overline{u\overline{u\overline{vw}}}$  +  $\frac{3}{2}\overline{u\overline{u\overline{vv}}}$  +  $\overline{u\overline{u\overline{wv}}}$  -  $\overline{u\overline{v\overline{uw}}}$  -
LS[u, - $\overline{uv} - \overline{uw}$ , - $\overline{u\overline{uv}}$  +  $\frac{1}{2}\overline{u\overline{vv}}$  +  $\overline{u\overline{wv}}$  +  $\frac{1}{2}\overline{u\overline{ww}}$ , - $\overline{u\overline{u\overline{uv}}}$  -  $\overline{u\overline{u\overline{vw}}}$  +  $\frac{3}{2}\overline{u\overline{u\overline{vv}}}$  +  $\overline{u\overline{u\overline{wv}}}$  -  $\overline{u\overline{v\overline{uw}}}$  -
True

```

# Testing the Conjugation Relation

```

TestingConjugationRelation
$SeriesShowDegree = 3;
{
  t1 = R+[u, x] ∪ R+[v, y] ∪ R+[w, z],
  t2 = t1 // tm[v, w, v],
  t3 = t2 // hm[x, y, x],
  t4 = t3 // tha[u, z],
  t5 = R+[v, x] ∪ R+[w, z] ∪ R+[u, y],
  t6 = t5 // tm[v, w, v],
  t7 = t6 // hm[x, y, x],
  t7 ≡ t4
} // ColumnForm

```

## Testing JA

```
$SeriesShowDegree = 3;
$SeriesCompareDegree = 6;
{
  {u = {"u"}, v = {"v"}, λ0 = MakeLieSeries[u + v + b[u, v]]},
  t1 = JA[u, (s + ε) λ0],
  t2 = JA[u, s λ0] // RC[u, ε λ0] // RC[u, s λ0]],
  t3 = JA[u, ε λ0] // RC[u, s λ0]],
  t1 ≈ t2 + t3
} // ColumnForm
{⟨u⟩, ⟨v⟩, LS[u + v, √uv, 0]}
CWS[s CW[u] + ε CW[u], -s CW[uv] + 1/2 s² CW[uv] - ε CW[uv] + s ε CW[uv] + 1/2 ε² CW[uv], 1/2 s² CW[uu]
CWS[s CW[u], -s CW[uv] + 1/2 s² CW[uv], 1/2 s² CW[uuv] - 1/6 s³ CW[uuv] - 1/2 s² CW[uvv] + 1/6 s³ CW[uvv]
CWS[ε CW[u], -ε CW[uv] + s ε CW[uv] + 1/2 ε² CW[uv], s ε CW[uuv] - 1/2 s² ε CW[uuv] + 1/2 ε² CW[uuv] -
True
```

## Testing J

```
$SeriesShowDegree = 6;
{{u = {"u"}, v = {"v"}, w = {"w"}},
 λx = MakeLieSeries[v + b[u, v] + b[u, b[u, v]]],
 div[u, λ // RC[u, s λx]] // LieMorphism[u → Ad[-s λx][u]]
}
{{⟨u⟩, ⟨v⟩, ⟨w⟩}, LS[v, √uv, √uuv, 0, 0, 0], CWS[CW[u], s CW[uv],
s CW[uuv] + 1/2 s² CW[uvv], s CW[uuuv] + 1/2 s² CW[uuvv] + s² CW[uvuv] + 1/6 s³ CW[uvvv],
1/2 s² CW[uuuvv] + 2 s² CW[uuvuv] + 1/6 s³ CW[uuvvv] + s³ CW[uvuvv] + 1/24 s⁴ CW[uvvvv],
1/2 s² CW[uuuuvuv] + 1/6 s³ CW[uuuvvvv] + 3/2 s² CW[uuvuuv] +
s³ CW[uuvuvvv] + 2/3 s³ CW[uuvvvuv] + 1/24 s⁴ CW[uuvvvvv] + 4/3 s³ CW[uvuvuv] +
1/3 s⁴ CW[uvuvvvv] + 1/4 s⁴ CW[uvvuvvv] + 1/120 s⁵ CW[uvvvvvv]]}
```

```

$SeriesShowDegree = 6;
$SeriesCompareDegree = 6;
{{t = <"t">, u = <"u">, v = <"v">, w = <"w">, uvw = LieMorphism[u → w, v → w]}, 
 λx = MakeLieSeries[v + b[u, v] + b[u, b[t, v]]], 
 t1 = J[w, λx // uvw] // RC[w, λx // uvw], 
 t2 = J[u, λx] // RC[u, λx] // RC[v, λx // RC[u, λx]] // uvw, 
 t3 = J[v, λx // RC[u, λx]] // RC[v, λx // RC[u, λx]] // uvw, 
 t1 ≡ t2 + t3
} // ColumnForm

{<t>, <u>, <v>, <w>, LieMorphism[LieMorphism$77999]}

LS[v,  $\overline{uv}$ ,  $\overline{tvu}$ , 0, 0, 0]
CWS[CW[w], 0, CW[tww], - $\frac{CW[twww]}{2}$ ,  $\frac{CW[twwww]}{6}$ ,  $\frac{CW[ttwwww]}{2}$  - CW[twtwww] -  $\frac{CW[twwwww]}{24}$ ]
CWS[0, -CW[ww], - $\frac{CW[www]}{2}$ , - $\frac{2CW[wwww]}{3}$ , - $\frac{13CW[wwwww]}{24}$ ,  $\frac{CW[ttwwww]}{2}$  - CW[twtwww] +  $\frac{CW[twwtww]}{2}$  -  $\frac{19CW[wwww]}{30}$ 
CWS[CW[w], CW[ww], CW[tww] +  $\frac{CW[www]}{2}$ , - $\frac{CW[twww]}{2}$  +  $\frac{2CW[wwww]}{3}$ ,  $\frac{CW[twwww]}{6}$  +  $\frac{13CW[wwwww]}{24}$ , - $\frac{CW[twwtww]}{2}$  - CI
True

$SeriesShowDegree = 3;
$SeriesCompareDegree = 6;
{{u = <"u">, v = <"v">, w = <"w">}, 
 λx = MakeLieSeries[v + b[u, v] + b[u, b[u, v]]], 
 λy = MakeLieSeries[w + b[u, b[v, w]]], 
 t1 = J[u, BCH[λx, λy]] // RC[u, λx], 
 t2 = J[u, λx] // RC[u, λx], 
 t3 = J[u, λy // RC[u, λx]], 
 t1 ≡ t2 + t3
} // ColumnForm

{<u>, <v>, <w>}
LS[v,  $\overline{uv}$ ,  $\overline{u\overline{uv}}$ ]
LS[w, 0,  $\overline{u\overline{vw}}$ ]
CWS[0, -CW[uv], -CW[uuv] -  $\frac{CW[uvv]}{2}$  - CW[uvw] + CW[uwv]]
CWS[0, -CW[uv], -CW[uuv] -  $\frac{CW[uvv]}{2}$ ]
CWS[0, 0, -CW[uvw] + CW[uwv]]
True

```

## Testing the relationship between $J$ and $JA$

```
$SeriesShowDegree = 3;
$SeriesCompareDegree = 7;
{{u = <"u">, v = <"v">},
 λ0 = MakeLieSeries[v + b[u, v] + b[u, b[u, v]]],
 t1 = J[u, λ0],
 t2 = JA[u, λ0] // LieMorphism[u → Ad[-λ0][u]],
 t1 ≡ t2
} // ColumnForm

{⟨u⟩, ⟨v⟩}
LS[v, uv, uuv]
CWS[0, -CW[uv], -CW[uuv] - CW[uuv]/2]
CWS[0, -CW[uv], -CW[uuv] - CW[uuv]/2]
True
```

## Computing $8_{17}$ .

```
μ0 = R-[12, 1] ∪ R-[2, 7] ∪ R-[8, 3] ∪
     R-[4, 11] ∪ R+[16, 5] ∪ R+[6, 13] ∪ R+[14, 9] ∪ R+[10, 15]

M[{1 → LS[-b, 0, 0], 2 → LS[0, 0, 0], 3 → LS[-8, 0, 0], 4 → LS[0, 0, 0],
  5 → LS[f, 0, 0], 6 → LS[0, 0, 0], 7 → LS[-2, 0, 0], 8 → LS[0, 0, 0], 9 → LS[d, 0, 0],
  10 → LS[0, 0, 0], 11 → LS[-4, 0, 0], 12 → LS[0, 0, 0], 13 → LS[6, 0, 0],
  14 → LS[0, 0, 0], 15 → LS[0, 0, 0], 16 → LS[0, 0, 0}], CWS[0, 0, 0]]

μ0 // dm[1, 2, 1]

M[{1 → LS[-b, 0, 0], 3 → LS[-8, 0, 0], 4 → LS[0, 0, 0], 5 → LS[f, 0, 0],
  6 → LS[0, 0, 0], 7 → LS[-1, 0, 0], 8 → LS[0, 0, 0], 9 → LS[d, 0, 0],
  10 → LS[0, 0, 0], 11 → LS[-4, 0, 0], 12 → LS[0, 0, 0], 13 → LS[6, 0, 0],
  14 → LS[0, 0, 0], 15 → LS[0, 0, 0], 16 → LS[0, 0, 0}], CWS[0, 0, 0]]

{
  μ0 = R-[12, 1] ∪ R-[2, 7] ∪ R-[8, 3] ∪
       R-[4, 11] ∪ R+[16, 5] ∪ R+[6, 13] ∪ R+[14, 9] ∪ R+[10, 15],
  Do[μ0 = μ0 // dm[1, k, 1], {k, 2, 16}]; μ0
} // ColumnForm

M[{1 → LS[-b, 0, 0], 2 → LS[0, 0, 0], 3 → LS[-8, 0, 0], 4 → LS[0, 0, 0], 5 → LS[f, 0, 0], 6 →
  M[{1 → LS[0, 0, 0}], CWS[0, -CW[11], 0]}]

μ0[[2]][{6}]

CWS[0, -CW[11], 0, -31 CW[1111]/12, 0, -1351 CW[111111]/360]
```

```

Print/@{ $\beta = -\frac{1 - 4 x + 8 x^2 - 11 x^3 + 8 x^4 - 4 x^5 + x^6}{x^3}$  /. x → ex,  

Series[ $\beta$ , {x, 0, 8}],  

Series[Log[ $\beta$ ], {x, 0, 9}]  
};


$$\begin{aligned} & -e^{-3x} \left( 1 - 4 e^x + 8 e^{2x} - 11 e^{3x} + 8 e^{4x} - 4 e^{5x} + e^{6x} \right) \\ & 1 - x^2 - \frac{25 x^4}{12} - \frac{481 x^6}{360} - \frac{1109 x^8}{4032} + O[x]^9 \\ & -x^2 - \frac{31 x^4}{12} - \frac{1351 x^6}{360} - \frac{123271 x^8}{20160} + O[x]^{10} \end{aligned}$$


```

## The Borromean Link

```

(* {r=<r>, g=<g>, b=<b>}; *)
μ0 = R-[r, 6] ∪ R+[2, 4] ∪ R-[g, 9] ∪ R+[5, 7] ∪ R-[b, 3] ∪ R+[8, 1]

M[{1 → LS[8, 0, 0], 2 → LS[0, 0, 0], 3 → LS[-b, 0, 0], 4 → LS[2, 0, 0],
5 → LS[0, 0, 0], 6 → LS[-r, 0, 0], 7 → LS[5, 0, 0], 8 → LS[0, 0, 0],
9 → LS[-g, 0, 0], b → LS[0, 0, 0], g → LS[0, 0, 0], r → LS[0, 0, 0]}, CWS[0, 0, 0]]

(1 /. μ0[[1]])[1] // FullForm
LW["8"]

μ0 // dm[r, 1, r]

M[ {2 → LS[0, 0, 0], 3 → LS[-b, 0, 0], 4 → LS[2, 0, 0], 5 → LS[0, 0, 0],
6 → LS[-r, -8r, -1/2 88r], 7 → LS[5, 0, 0], 8 → LS[0, 0, 0], 9 → LS[-g, 0, 0],
b → LS[0, 0, 0], g → LS[0, 0, 0], r → LS[8, 0, 0]}, CWS[0, 0, 0] ]

Do[μ0 = μ0 // dm[r, k, r], {k, 1, 3}];
Do[μ0 = μ0 // dm[g, k, g], {k, 4, 6}];
Do[μ0 = μ0 // dm[b, k, b], {k, 7, 9}];
μ0

M[ {b → LS[0, gr, 1/2 gggr + brg + 1/2 grr], g → LS[0, -br, 1/2 bbr - bggr - brg + 1/2 brr],
r → LS[0, bg, 1/2 bbgb + bggr + 1/2 bgg]}, CWS[0, 0, 2 CW[bgr]] ]

μ1 = Sort/@((LieMorphism[<r> → <g>, <g> → <b>, <b> → <r>] /@ μ0) /.
Thread[{r, g, b} → {g, b, r}])

M[ {b → LS[0, gr, 1/2 gggr + brg + 1/2 grr], g → LS[0, -br, 1/2 bbr - bggr - brg + 1/2 brr],
r → LS[0, bg, 1/2 bbgb + bggr + 1/2 bgg]}, CWS[0, 0, 2 CW[bgr]] ]

```

```

μ2 = Sort /@ ((LieMorphism[⟨r⟩ → ⟨g⟩, ⟨g⟩ → ⟨r⟩] /@ μ0) /. Thread[{r, g} → {g, r}])
M[{b → LS[0, -g r, b g r + 1/2 g g r + b r g + 1/2 g r r], g → LS[0, b r, 1/2 b b r - b g r + 1/2 b r r],
r → LS[0, -b g, 1/2 b b g + 1/2 b g g - b r g]}, CWS[0, 0, 2 CW[b r g]]]

$SeriesShowDegree = 4; μ0
M[{b → LS[0, g r, 1/2 g g r + b r g + 1/2 g r r,
-1/2 b b r g + 1/6 g g g r + 1/4 g g r r - 1/2 b g b r - 1/2 b r g g - 1/2 b r r g + 1/6 g r r r], g →
LS[0, -b r, 1/2 b b r - b g r - b r g + 1/2 b r r, -1/6 b b b r - 1/2 b b g r - 1/2 b g g r - 1/2 b b r g -
1/4 b b r r + 1/2 b g r r + 1/2 b g b r + b r g r - b g r g - 1/2 b r g g + 1/2 b r r g - 1/6 b r r r],
r → LS[0, b g, 1/2 b b g + b g r + 1/2 b g g, 1/6 b b b g + 1/2 b b g r +
1/2 b g g r + 1/4 b b g g + 1/2 b g r r + 1/6 b g g g]}, CWS[0, 0, 2 CW[b g r], CW[bb g r] - CW[b g b r] + CW[b g g r] - CW[b g r g] + CW[b g r r] - CW[b r g r]]]

$SeriesShowDegree = 4; μ1
M[{b → LS[0, g r, 1/2 g g r + b r g + 1/2 g r r,
-1/2 b b r g + 1/6 g g g r + 1/4 g g r r - 1/2 b g b r - 1/2 b r g g - 1/2 b r r g + 1/6 g r r r], g →
LS[0, -b r, 1/2 b b r - b g r - b r g + 1/2 b r r, -1/6 b b b r - 1/2 b b g r - 1/2 b g g r - 1/2 b b r g -
1/4 b b r r + 1/2 b g r r + 1/2 b g b r + b r g r - b g r g - 1/2 b r g g + 1/2 b r r g - 1/6 b r r r],
r → LS[0, b g, 1/2 b b g + b g r + 1/2 b g g, 1/6 b b b g + 1/2 b b g r +
1/2 b g g r + 1/4 b b g g + 1/2 b g r r + 1/6 b g g g]}, CWS[0, 0, 2 CW[b g r], CW[bb g r] - CW[b g b r] + CW[b g g r] - CW[b g r g] + CW[b g r r] - CW[b r g r]]]

```

**\$SeriesShowDegree = 4;  $\mu_2$**

$$\begin{aligned} M\left[ \left\{ b \rightarrow \right. \right. \\ LS\left[ 0, -\overline{gr}, \frac{1}{2}\overline{ggr} + \frac{1}{2}\overline{brg} + \frac{1}{2}\overline{grr}, -\frac{1}{2}\overline{bbgr} - \frac{1}{2}\overline{bgrg} - \frac{1}{2}\overline{bbrg} - \frac{1}{2}\overline{bgrr} - \right. \\ \left. \left. \frac{1}{6}\overline{gggr} - \frac{1}{4}\overline{ggrr} + \frac{1}{2}\overline{bgbr} - \overline{brgr} - \overline{bgrg} - \frac{1}{2}\overline{brgg} - \frac{1}{2}\overline{brrg} - \frac{1}{6}\overline{grrr} \right], \right. \\ g \rightarrow LS\left[ 0, \overline{br}, \frac{1}{2}\overline{bbr} - \overline{bgr} + \frac{1}{2}\overline{brr}, \frac{1}{6}\overline{bbb} - \frac{1}{2}\overline{bgr} + \frac{1}{2}\overline{bggr} + \right. \\ \left. \left. \frac{1}{4}\overline{bbr} + \frac{1}{2}\overline{bgr} + \frac{1}{6}\overline{brrr} \right], r \rightarrow LS\left[ 0, -\overline{bg}, \frac{1}{2}\overline{bbg} + \frac{1}{2}\overline{bgg} - \overline{brg}, \right. \\ \left. \left. -\frac{1}{6}\overline{bbb} - \frac{1}{4}\overline{bfg} - \frac{1}{2}\overline{bbg} - \frac{1}{2}\overline{bgbr} - \frac{1}{6}\overline{bggg} + \frac{1}{2}\overline{brgg} - \frac{1}{2}\overline{brrg} \right] \right], \\ CWS[0, 0, 2 CW[brg], CW[bbrg] - CW[bgrb] - CW[bgrg] + CW[brgg] - CW[brgr] + CW[brrg]] \end{aligned}$$

**\$SeriesShowDegree = 5;  $\mu_0$**

$$\begin{aligned} M\left[ \left\{ b \rightarrow LS\left[ 0, \overline{gr}, \frac{1}{2}\overline{ggr} + \overline{brg} + \frac{1}{2}\overline{grr}, \right. \right. \right. \\ \left. \left. \left. -\frac{1}{2}\overline{bbgr} + \frac{1}{6}\overline{gggr} + \frac{1}{4}\overline{ggrr} - \frac{1}{2}\overline{bgbr} - \frac{1}{2}\overline{brgg} - \frac{1}{2}\overline{brrg} + \frac{1}{6}\overline{grrr}, \right. \right. \\ \left. \left. \frac{1}{6}\overline{bbbrg} + \frac{1}{6}\overline{bbgb} + \frac{1}{4}\overline{bbrg} + \frac{1}{4}\overline{bbr} + \frac{1}{24}\overline{gggr} + \right. \right. \\ \left. \left. \frac{1}{12}\overline{gggr} + \frac{1}{12}\overline{ggrr} + \frac{1}{2}\overline{bgbr} + \frac{1}{4}\overline{bgbr} - \frac{3}{2}\overline{brgg} - \right. \right. \\ \left. \left. \frac{1}{2}\overline{brbr} - \frac{1}{6}\overline{bbrb} - \overline{bgrgr} - 2\overline{bgrrg} - \frac{1}{12}\overline{ggrgr} + \frac{1}{4}\overline{bggb} - \right. \right. \\ \left. \left. \frac{9}{2}\overline{brgrg} + \frac{1}{6}\overline{brgg} - \frac{3}{4}\overline{brrgg} + \frac{1}{6}\overline{brrrg} + \frac{1}{24}\overline{grrrr} \right], g \rightarrow \right. \\ LS\left[ 0, -\overline{br}, \frac{1}{2}\overline{bbr} - \overline{bgr} - \overline{brg} + \frac{1}{2}\overline{brr}, -\frac{1}{6}\overline{bbb} - \frac{1}{2}\overline{bgr} - \frac{1}{2}\overline{bggr} - \frac{1}{2}\overline{bbrg} - \right. \\ \left. \left. \frac{1}{4}\overline{bbr} + \frac{1}{2}\overline{bgr} + \frac{1}{2}\overline{bgbr} + \overline{brgr} - \overline{bgrg} - \frac{1}{2}\overline{brgg} + \frac{1}{2}\overline{brrg} - \frac{1}{6}\overline{brrr}, \right. \right. \\ \left. \left. \frac{1}{24}\overline{bbbr} - \frac{1}{6}\overline{bbbg} - \frac{1}{4}\overline{bbgr} - \frac{1}{6}\overline{bbbr} + \frac{1}{12}\overline{bbbr} - \frac{3}{4}\overline{bbgr} - \frac{1}{6}\overline{bggg} + \right. \right. \\ \left. \left. \frac{1}{4}\overline{bggr} + \frac{1}{3}\overline{bbgb} + \frac{1}{2}\overline{bbrgr} - \frac{1}{2}\overline{bbrgr} - \frac{1}{4}\overline{bbgrg} + \frac{5}{4}\overline{bbrrg} + \frac{1}{12}\overline{bbrrr} - \right. \right. \\ \left. \left. \frac{1}{6}\overline{bgrrr} - \frac{1}{2}\overline{bgbr} - \frac{1}{2}\overline{bgbr} - \frac{1}{4}\overline{bgbr} + \frac{1}{2}\overline{brgg} + 2\overline{brbrg} - \frac{1}{2}\overline{brgr} + \right. \right. \\ \left. \left. \frac{1}{12}\overline{bbrbg} - \frac{1}{12}\overline{bbrbr} - \overline{bgrbr} + \frac{1}{2}\overline{bgrgr} - \frac{1}{2}\overline{bggrg} + \frac{1}{2}\overline{bgrrg} + \frac{1}{4}\overline{bggb} - \right. \right. \\ \left. \left. \frac{1}{2}\overline{brrgr} + \overline{brgrg} - \frac{1}{2}\overline{bgrrg} - \frac{1}{6}\overline{brgg} + \frac{1}{4}\overline{brrg} - \frac{1}{6}\overline{brrg} + \frac{1}{24}\overline{brrrr} \right], \right. \end{aligned}$$

$$\begin{aligned}
r \rightarrow LS & \left[ 0, \overline{bg}, \frac{1}{2} \overline{\overline{bbg}} + \overline{b\overline{gr}}, \frac{1}{2} \overline{\overline{b\overline{gg}}}, \frac{1}{6} \overline{\overline{\overline{bbg}}}, \frac{1}{2} \overline{\overline{b\overline{b\overline{bg}}}} + \frac{1}{2} \overline{\overline{b\overline{b\overline{gr}}}} + \frac{1}{2} \overline{\overline{b\overline{g\overline{gr}}}} + \frac{1}{4} \overline{\overline{\overline{b\overline{b\overline{gg}}}}} + \right. \\
& \frac{1}{2} \overline{\overline{b\overline{g\overline{rr}}}}, \frac{1}{6} \overline{\overline{b\overline{g\overline{g}}}}, \frac{1}{24} \overline{\overline{\overline{bb\overline{bbg}}}}, \frac{1}{6} \overline{\overline{b\overline{b\overline{b\overline{gr}}}}}, \frac{1}{4} \overline{\overline{b\overline{b\overline{g\overline{gr}}}}}, \frac{1}{12} \overline{\overline{b\overline{b\overline{b\overline{g\overline{g}}}}}} + \\
& \frac{1}{4} \overline{\overline{b\overline{b\overline{g\overline{rr}}}}}, \frac{1}{6} \overline{\overline{b\overline{g\overline{g\overline{r}}}}}, \frac{1}{4} \overline{\overline{b\overline{g\overline{g\overline{rr}}}}}, - \overline{b\overline{b\overline{g\overline{r}}}}, \frac{1}{12} \overline{\overline{b\overline{b\overline{g\overline{g}}}}}, - 2 \overline{b\overline{b\overline{r\overline{g}}}}, + \\
& \left. \frac{1}{6} \overline{\overline{b\overline{g\overline{r\overline{r}}}}}, \frac{1}{2} \overline{\overline{b\overline{g\overline{b\overline{gr}}}}}, - \overline{b\overline{g\overline{b\overline{rg}}}}, - \frac{1}{12} \overline{\overline{b\overline{b\overline{g\overline{b\overline{g}}}}}}, - \frac{1}{2} \overline{\overline{b\overline{g\overline{r\overline{g\overline{r}}}}}}, + \frac{1}{24} \overline{\overline{\overline{b\overline{g\overline{g\overline{g\overline{g}}}}}}} \right\}, \\
CWS & \left[ 0, 0, 2 CW[bgr], CW[bbgr] - CW[bgb], CW[bgr] - CW[bgrg] + CW[bgrr] - CW[brgr], \right. \\
& \frac{CW[bbbgr]}{3} - \frac{CW[bbgbr]}{2} + \frac{CW[bbggr]}{2} + \frac{CW[bbgrg]}{2} + \\
& \frac{CW[bbgrr]}{2} + \frac{CW[bbrbg]}{2} - \frac{3 CW[bbrgr]}{2} + \frac{CW[bgbrr]}{2} - \\
& \frac{3 CW[bggbr]}{2} + \frac{CW[bgggr]}{3} - \frac{CW[bggrg]}{2} + \frac{CW[bggrr]}{2} + \frac{CW[bgrgg]}{2} - \\
& \left. \frac{3 CW[bgrrg]}{2} + \frac{CW[bgrrr]}{3} + \frac{CW[brggr]}{2} - \frac{CW[brgrr]}{2} + \frac{CW[brrgr]}{2} \right]
\end{aligned}$$

**(r /. First[μ0])[[5]]**

$$\begin{aligned}
LS & \left[ 0, \langle b\overline{g} \rangle, \frac{\langle b\overline{bg} \rangle}{2} + \frac{\langle b\overline{gg} \rangle}{2} + \langle b\overline{gr} \rangle, \frac{\langle b\overline{bbg} \rangle}{6} + \frac{\langle b\overline{b\overline{gg}} \rangle}{4} + \frac{\langle b\overline{b\overline{gr}} \rangle}{2} + \frac{\langle b\overline{g\overline{gg}} \rangle}{6} + \frac{\langle b\overline{g\overline{gr}} \rangle}{2} + \frac{\langle b\overline{g\overline{rr}} \rangle}{2}, \right. \\
& \frac{\langle b\overline{bbb\overline{g}} \rangle}{24} + \frac{\langle b\overline{bb\overline{gg}} \rangle}{12} + \frac{\langle b\overline{bb\overline{gr}} \rangle}{6} - \frac{\langle b\overline{b\overline{b\overline{bg}} \rangle}}{12} + \frac{\langle b\overline{b\overline{b\overline{gg}} \rangle}}{12} + \frac{\langle b\overline{b\overline{b\overline{gr}} \rangle}}{4} - \langle b\overline{b\overline{g\overline{rg}} \rangle} + \frac{\langle b\overline{b\overline{g\overline{rr}} \rangle}}{4} - \\
& \left. 2 \langle b\overline{b\overline{rg\overline{g}} \rangle} + \frac{\langle b\overline{g\overline{b\overline{gr}} \rangle}}{2} - \langle b\overline{g\overline{b\overline{rg}} \rangle} + \frac{\langle b\overline{g\overline{gg\overline{g}} \rangle}}{24} + \frac{\langle b\overline{g\overline{gg\overline{r}} \rangle}}{6} + \frac{\langle b\overline{g\overline{gr\overline{r}} \rangle}}{4} - \frac{\langle b\overline{g\overline{r\overline{gr}} \rangle}}{2} + \frac{\langle b\overline{g\overline{rr\overline{r}} \rangle}}{6} \right]
\end{aligned}$$

**\$SeriesShowDegree = 6; Last[\mu\_0]**

$$\begin{aligned}
 & \text{CWS}\left[0, 0, 2 \text{CW[bgr]}, \text{CW[bbgr]} - \text{CW[bgbr]} + \text{CW[bggr]} - \text{CW[bgrg]} + \text{CW[bgrr]} - \text{CW[brgr]}, \right. \\
 & \frac{\text{CW[bbbgr]} - \text{CW[bbgbr]} + \text{CW[bbggr]} + \text{CW[bbgrg]} + \text{CW[bbgrr]} + \text{CW[bbrbg]}}{3} - \\
 & \frac{3 \text{CW[bbngr]} + \text{CW[bgbr]} - 3 \text{CW[bggbr]} + \text{CW[bgggr]} - \text{CW[bggrg]} + \text{CW[bggrr]}}{2} + \\
 & \frac{\text{CW[bgrrg]} - 3 \text{CW[bgrrg]} + \text{CW[bgrrr]} + \text{CW[brgrg]} - \text{CW[brgrr]} + \text{CW[brrgr]}}{2}, \\
 & \text{CW[bbbbgr]} - \frac{\text{CW[bbbgr]} + \text{CW[bbbgrr]} - \text{CW[bbbgrg]} + \text{CW[bbbgrr]} - \text{CW[bbrbg]}}{12} - \\
 & \frac{\text{CW[bbbrgr]} + \text{CW[bbgbbr]} + \text{CW[bbgbrr]} - 3 \text{CW[bbggbr]} + \text{CW[bbgggr]} + \text{CW[bbgggrg]}}{6} + \\
 & \frac{\text{CW[bbggrr]} + \text{CW[bbgrbg]} - \text{CW[bbgrgg]} + \text{CW[bbgrgr]} - 5 \text{CW[bbgrrg]} + \text{CW[bbgrrr]}}{4} + \\
 & \frac{3 \text{CW[bbrbgg]} + \text{CW[bbrgbr]} - 5 \text{CW[bbrggr]} + \text{CW[bbrgrg]} - 3 \text{CW[bbrgrr]}}{4} - \\
 & \frac{\text{CW[bbrrbg]} + 3 \text{CW[bbrrgr]} - \text{CW[bgbgbr]} - \text{CW[bgbggr]} + \text{CW[bgbgrr]} - \text{CW[bgbrbr]}}{4} + \\
 & \text{CW[bgbrrg]} - \frac{\text{CW[bgbrrr]} + \text{CW[bggbrg]} - 5 \text{CW[bggbrr]} - \text{CW[bgggbr]} + \text{CW[bggggr]}}{6} - \\
 & \frac{\text{CW[bggggr]} + \text{CW[bgggrr]} + \text{CW[bggrbr]} + \text{CW[bggrgg]} - 3 \text{CW[bggrrg]} + \text{CW[bggrrr]}}{6} - \\
 & 4 \text{CW[bgrbgr]} + \text{CW[bgrbrr]} - \frac{\text{CW[bgrggg]}}{6} + \text{CW[bgrggr]} - \text{CW[bgrgrg]} + \frac{3 \text{CW[bgrrgg]}}{4} - \\
 & \frac{\text{CW[bgrrrg]} + \text{CW[bgrrrr]} + \text{CW[brbrgg]} - \text{CW[brbrgr]} - \text{CW[brgggr]} + \text{CW[brggrr]}}{6} - \\
 & \left. \text{CW[brgrgr]} + \text{CW[brgrrg]} - \frac{\text{CW[brgrrr]}}{6} - \frac{\text{CW[brrggr]}}{4} + \frac{\text{CW[brrgrr]}}{4} - \frac{\text{CW[brrrgr]}}{6}\right]
 \end{aligned}$$

**\$SeriesShowDegree = 7; r /. First[\mu\_0]**

$$\begin{aligned}
 & \text{LS}\left[0, \overline{b}\overline{g}, \frac{1}{2}\overline{\overline{b}\overline{b}\overline{g}} + \overline{b}\overline{g}\overline{r} + \frac{1}{2}\overline{\overline{b}\overline{g}\overline{g}}, \right. \\
 & \frac{1}{6}\overline{\overline{b}\overline{b}\overline{b}\overline{g}} + \frac{1}{2}\overline{\overline{b}\overline{b}\overline{g}\overline{r}} + \frac{1}{2}\overline{\overline{b}\overline{g}\overline{g}\overline{r}} + \frac{1}{4}\overline{\overline{b}\overline{b}\overline{g}\overline{g}} + \frac{1}{2}\overline{\overline{b}\overline{g}\overline{r}\overline{r}} + \frac{1}{6}\overline{\overline{b}\overline{g}\overline{g}\overline{g}}, \\
 & \frac{1}{24}\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{g}}} + \frac{1}{6}\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{g}\overline{r}}} + \frac{1}{4}\overline{\overline{\overline{b}\overline{b}\overline{g}\overline{g}\overline{r}}} + \frac{1}{12}\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{g}\overline{g}}} + \frac{1}{4}\overline{\overline{\overline{b}\overline{b}\overline{g}\overline{r}\overline{r}}} + \\
 & \frac{1}{6}\overline{\overline{\overline{b}\overline{g}\overline{g}\overline{g}\overline{r}}} + \frac{1}{4}\overline{\overline{\overline{b}\overline{g}\overline{g}\overline{r}\overline{r}}} - \overline{\overline{b}\overline{g}\overline{r}\overline{g}\overline{g}} + \frac{1}{12}\overline{\overline{\overline{b}\overline{b}\overline{g}\overline{g}\overline{g}}} - 2\overline{\overline{b}\overline{b}\overline{r}\overline{g}\overline{g}} + \frac{1}{6}\overline{\overline{\overline{b}\overline{g}\overline{r}\overline{r}\overline{r}}}, \\
 & \frac{1}{2}\overline{\overline{\overline{b}\overline{g}\overline{b}\overline{g}\overline{r}}} - \overline{\overline{b}\overline{g}\overline{b}\overline{r}\overline{g}} - \frac{1}{12}\overline{\overline{\overline{b}\overline{b}\overline{g}\overline{b}\overline{g}}} - \frac{1}{2}\overline{\overline{\overline{b}\overline{g}\overline{r}\overline{g}\overline{r}}} + \frac{1}{24}\overline{\overline{\overline{b}\overline{g}\overline{g}\overline{g}\overline{g}}}, \\
 & \left. \frac{1}{120}\overline{\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{b}\overline{b}\overline{g}}}} + \frac{1}{24}\overline{\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{b}\overline{g}\overline{r}}}} + \frac{1}{12}\overline{\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{g}\overline{g}\overline{r}}}} + \frac{1}{48}\overline{\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{b}\overline{g}\overline{g}}}} + \frac{1}{12}\overline{\overline{\overline{\overline{b}\overline{b}\overline{b}\overline{g}\overline{r}\overline{r}}}} + \right.
 \end{aligned}$$

$$\begin{aligned}
& \frac{1}{12} \overbrace{\text{b b g g g r}} + \frac{1}{8} \overbrace{\text{b b g g r r}} - \frac{1}{36} \overbrace{\text{b b b g g r g}} + \frac{1}{12} \overbrace{\text{b b b g g g - b b b r g r g}} + \frac{1}{12} \overbrace{\text{b b g r r r}} \\
& \frac{1}{24} \overbrace{\text{b g g g g g r}} + \frac{1}{12} \overbrace{\text{b g g g r r r}} + \frac{1}{12} \overbrace{\text{b g g r r r r}} - \frac{1}{4} \overbrace{\text{b b g b g r}} - \frac{1}{24} \overbrace{\text{b b g b g g}} \\
& \frac{1}{2} \overbrace{\text{b b b g b r g}} - \frac{1}{24} \overbrace{\text{b b b r g g r}} - \frac{1}{4} \overbrace{\text{b b b g b g}} - \frac{5}{4} \overbrace{\text{b b g r g r}} - \frac{1}{2} \overbrace{\text{b b g g r g}} \\
& \frac{3}{2} \overbrace{\text{b b g r r g}} - \frac{1}{4} \overbrace{\text{b g r g r r}} - \frac{1}{3} \overbrace{\text{b g g r g r}} + \frac{1}{2} \overbrace{\text{b b g g b r}} - 4 \overbrace{\text{b b b r g r g}} - \frac{3}{2} \overbrace{\text{b b g r g g}} \\
& \frac{1}{48} \overbrace{\text{b b g g g g - b b r g g g}} + \frac{1}{24} \overbrace{\text{b g r r r r}} + \frac{1}{4} \overbrace{\text{b g b g g r}} + \frac{1}{4} \overbrace{\text{b g b g r r}} + \frac{1}{2} \overbrace{\text{b g b g b r}} \\
& \overbrace{\text{b g b g r g}} - \frac{3}{2} \overbrace{\text{b g b r g g}} + \frac{1}{2} \overbrace{\text{b g b r r g}} - \frac{1}{2} \overbrace{\text{b g r g g r}} - \overbrace{\text{b g r b r g}} - \frac{1}{12} \overbrace{\text{b b g g b g}} \\
& \overbrace{\text{b b r g b g}} - \frac{1}{2} \overbrace{\text{b g r r g r}} + \frac{1}{4} \overbrace{\text{b g g b g r}} - \frac{1}{2} \overbrace{\text{b g g b r g}} + \frac{1}{120} \overbrace{\text{b g g g g g g}}, \\
& \frac{1}{720} \overbrace{\text{b b b b b b g}} + \frac{1}{120} \overbrace{\text{b b b b b g r}} + \frac{1}{48} \overbrace{\text{b b b b g g r}} + \frac{1}{240} \overbrace{\text{b b b b b g g}} + \frac{1}{48} \overbrace{\text{b b b b g r r}} \\
& \frac{1}{36} \overbrace{\text{b b b g g g r}} + \frac{1}{24} \overbrace{\text{b b b g g r r}} - \frac{7}{12} \overbrace{\text{b b b b g r g}} + \frac{1}{144} \overbrace{\text{b b b b g g g}} - \frac{2}{3} \overbrace{\text{b b b b r g g}} \\
& \frac{1}{36} \overbrace{\text{b b b g r r r r}} + \frac{1}{48} \overbrace{\text{b b g g g g r}} + \frac{1}{24} \overbrace{\text{b b g g g r r}} + \frac{1}{24} \overbrace{\text{b b g g r r r r}} - \frac{1}{3} \overbrace{\text{b b b b g b g r}} \\
& \frac{1}{48} \overbrace{\text{b b b g b g g}} + \frac{1}{6} \overbrace{\text{b b b g b r g}} - \frac{1}{80} \overbrace{\text{b b b b g b g}} - \frac{13}{12} \overbrace{\text{b b b b g r g r}} \\
& \frac{1}{2} \overbrace{\text{b b b g g g r g}} - \frac{3}{2} \overbrace{\text{b b b g r r g}} - \frac{5}{8} \overbrace{\text{b b g r g r r}} - \frac{2}{3} \overbrace{\text{b b g g r g r}} + \frac{2}{3} \overbrace{\text{b b b g g b r}} - 3 \overbrace{\text{b b b b r g r g}} \\
& \frac{3}{2} \overbrace{\text{b b b g r g g g}} + \frac{1}{144} \overbrace{\text{b b b g g g g}} - \frac{1}{2} \overbrace{\text{b b b r g g g}} - \frac{1}{2} \overbrace{\text{b b b r r g g}} + \frac{1}{48} \overbrace{\text{b b g r r r r}} \\
& \frac{1}{120} \overbrace{\text{b g g g g g g r}} + \frac{1}{48} \overbrace{\text{b g g g g r r r}} + \frac{1}{36} \overbrace{\text{b g g g r r r r}} - \frac{1}{6} \overbrace{\text{b g g r g r r r}} - \frac{1}{8} \overbrace{\text{b g g g r g r r}} \\
& \frac{1}{48} \overbrace{\text{b g g r r r r r}} - \frac{1}{8} \overbrace{\text{b b g b g g r}} - \frac{5}{8} \overbrace{\text{b b g b g r r}} + \frac{1}{12} \overbrace{\text{b b g b g b r}} - 2 \overbrace{\text{b b g b g r g}} \\
& \frac{1}{72} \overbrace{\text{b b g b g g g}} - \frac{7}{4} \overbrace{\text{b b g b r g g}} - \frac{1}{4} \overbrace{\text{b b g b r r g}} - \frac{1}{4} \overbrace{\text{b b r g g g r}} - \overbrace{\text{b b r g g r r}} + \frac{1}{3} \overbrace{\text{b b b r b g g}} \\
& \frac{1}{6} \overbrace{\text{b b b g r b g}} - \frac{9}{4} \overbrace{\text{b b g r g g r}} + \frac{1}{2} \overbrace{\text{b b g r b r g}} + \overbrace{\text{b b g g r b r}} - \frac{1}{6} \overbrace{\text{b b g g g r g}} - \frac{3}{4} \overbrace{\text{b b g g r r g}}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{24} \overbrace{\text{bb}\text{bg}\text{g}\text{bg}} + \frac{2}{3} \overbrace{\text{bb}\text{b}\text{rg}\text{bg}} - \frac{9}{4} \overbrace{\text{bb}\text{g}\text{rr}\text{gr}} - \frac{7}{6} \overbrace{\text{bb}\text{g}\text{rr}\text{rg}} - \frac{1}{12} \overbrace{\text{b}\text{g}\text{r}\text{g}\text{rr}\text{r}} \\
& \frac{1}{3} \overbrace{\text{b}\text{g}\text{g}\text{r}\text{r}\text{gr}} - \frac{5}{8} \overbrace{\text{b}\text{b}\text{g}\text{g}\text{b}\text{gr}} - \frac{1}{4} \overbrace{\text{b}\text{b}\text{gg}\text{b}\text{rg}} - 2 \overbrace{\text{b}\text{b}\text{br}\text{g}\text{r}\text{gr}} - 2 \overbrace{\text{b}\text{b}\text{br}\text{g}\text{gr}\text{g}} - 3 \overbrace{\text{b}\text{b}\text{br}\text{g}\text{r}\text{rg}} + \\
& 2 \overbrace{\text{b}\text{b}\text{gr}\text{g}\text{br}} - \frac{3}{2} \overbrace{\text{b}\text{b}\text{gr}\text{g}\text{rg}} - \frac{3}{4} \overbrace{\text{b}\text{b}\text{gg}\text{gr}\text{gg}} + \frac{3}{4} \overbrace{\text{b}\text{b}\text{g}\text{rr}\text{gg}} + \frac{1}{2} \overbrace{\text{b}\text{b}\text{gg}\text{g}\text{br}} - \overbrace{\text{b}\text{b}\text{rr}\text{g}\text{rg}} + \\
& \overbrace{\text{b}\text{b}\text{br}\text{g}\text{rg}\text{gg}} - \frac{7}{6} \overbrace{\text{b}\text{b}\text{gr}\text{g}\text{gg}\text{g}} + \frac{1}{240} \overbrace{\text{b}\text{b}\text{gg}\text{g}\text{gg}\text{g}} - \frac{2}{3} \overbrace{\text{b}\text{b}\text{rg}\text{g}\text{gg}\text{g}} + \frac{1}{2} \overbrace{\text{b}\text{b}\text{rr}\text{g}\text{gg}\text{g}} - \frac{1}{3} \overbrace{\text{b}\text{b}\text{rr}\text{r}\text{gg}\text{g}} + \\
& \frac{1}{120} \overbrace{\text{b}\text{g}\text{rr}\text{r}\text{rr}\text{r}} + \frac{1}{12} \overbrace{\text{b}\text{g}\text{bg}\text{g}\text{gr}\text{r}} + \frac{1}{8} \overbrace{\text{b}\text{g}\text{bg}\text{g}\text{rr}\text{r}} + \frac{1}{12} \overbrace{\text{b}\text{g}\text{b}\text{g}\text{rr}\text{r}\text{r}} - \frac{5}{12} \overbrace{\text{b}\text{g}\text{bg}\text{b}\text{gr}\text{r}} + \\
& \frac{1}{2} \overbrace{\text{b}\text{g}\text{bg}\text{b}\text{rg}} - \frac{1}{4} \overbrace{\text{b}\text{g}\text{bg}\text{b}\text{rr}} + \frac{1}{2} \overbrace{\text{b}\text{g}\text{br}\text{b}\text{rg}} - \frac{1}{4} \overbrace{\text{b}\text{g}\text{b}\text{g}\text{r}\text{gr}} - \frac{1}{2} \overbrace{\text{b}\text{g}\text{b}\text{g}\text{gr}\text{g}} + \\
& \frac{1}{2} \overbrace{\text{b}\text{g}\text{b}\text{g}\text{rr}\text{g}} + \frac{3}{4} \overbrace{\text{b}\text{g}\text{b}\text{g}\text{g}\text{br}} + \overbrace{\text{b}\text{g}\text{b}\text{r}\text{g}\text{rg}} - \frac{3}{2} \overbrace{\text{b}\text{g}\text{b}\text{g}\text{r}\text{gg}} - \frac{7}{6} \overbrace{\text{b}\text{g}\text{b}\text{r}\text{g}\text{gg}} + \frac{3}{4} \overbrace{\text{b}\text{g}\text{b}\text{r}\text{r}\text{gg}} - \\
& \frac{1}{6} \overbrace{\text{b}\text{g}\text{b}\text{r}\text{r}\text{rg}} - \frac{1}{24} \overbrace{\text{b}\text{b}\text{g}\text{b}\text{b}\text{gr}} + \frac{7}{12} \overbrace{\text{b}\text{b}\text{g}\text{b}\text{b}\text{rg}} + \frac{1}{240} \overbrace{\text{b}\text{b}\text{b}\text{g}\text{b}\text{bg}} + \frac{1}{12} \overbrace{\text{b}\text{b}\text{g}\text{r}\text{b}\text{gr}} - \\
& \frac{1}{6} \overbrace{\text{b}\text{g}\text{r}\text{g}\text{gg}\text{gr}} - \frac{1}{4} \overbrace{\text{b}\text{g}\text{r}\text{g}\text{gr}\text{rr}} + \frac{1}{2} \overbrace{\text{b}\text{g}\text{r}\text{b}\text{rg}\text{g}} + \frac{1}{2} \overbrace{\text{b}\text{g}\text{r}\text{b}\text{rr}\text{g}} - \frac{1}{8} \overbrace{\text{b}\text{g}\text{g}\text{r}\text{g}\text{gr}} - \\
& \frac{1}{2} \overbrace{\text{b}\text{g}\text{g}\text{r}\text{b}\text{rg}} - \frac{1}{48} \overbrace{\text{b}\text{b}\text{gg}\text{b}\text{gg}} + \frac{5}{12} \overbrace{\text{b}\text{b}\text{gb}\text{b}\text{rg}} - \overbrace{\text{b}\text{b}\text{rg}\text{b}\text{gr}} - \frac{1}{2} \overbrace{\text{b}\text{b}\text{rg}\text{b}\text{gg}} + \\
& \frac{1}{2} \overbrace{\text{b}\text{b}\text{gr}\text{g}\text{bg}} - \frac{1}{4} \overbrace{\text{b}\text{g}\text{rr}\text{g}\text{gr}} - \frac{1}{2} \overbrace{\text{b}\text{g}\text{rr}\text{b}\text{rg}} - \frac{1}{8} \overbrace{\text{b}\text{g}\text{rr}\text{g}\text{rr}} - \frac{1}{36} \overbrace{\text{b}\text{b}\text{gg}\text{g}\text{bg}} + \\
& \frac{1}{2} \overbrace{\text{b}\text{b}\text{rg}\text{g}\text{bg}} - \frac{1}{2} \overbrace{\text{b}\text{b}\text{r}\text{g}\text{b}\text{g}} - \frac{1}{6} \overbrace{\text{b}\text{g}\text{rr}\text{r}\text{gr}} + \frac{1}{8} \overbrace{\text{b}\text{g}\text{g}\text{b}\text{gg}\text{r}} + \frac{1}{8} \overbrace{\text{b}\text{g}\text{g}\text{b}\text{gr}\text{r}} - \\
& \frac{1}{2} \overbrace{\text{b}\text{g}\text{g}\text{b}\text{gr}\text{g}} - \frac{3}{4} \overbrace{\text{b}\text{g}\text{g}\text{b}\text{rg}\text{g}} + \frac{1}{4} \overbrace{\text{b}\text{g}\text{g}\text{b}\text{rr}\text{g}} - \frac{1}{2} \overbrace{\text{b}\text{g}\text{b}\text{r}\text{b}\text{gr}} - \frac{1}{4} \overbrace{\text{b}\text{g}\text{b}\text{r}\text{b}\text{gg}} - \\
& \frac{1}{6} \overbrace{\text{b}\text{b}\text{r}\text{b}\text{g}\text{bg}} + \frac{1}{6} \overbrace{\text{b}\text{g}\text{r}\text{g}\text{gr}\text{gr}} + \frac{1}{12} \overbrace{\text{b}\text{g}\text{g}\text{g}\text{b}\text{gr}} - \frac{1}{6} \overbrace{\text{b}\text{g}\text{g}\text{g}\text{b}\text{rg}} + \frac{1}{720} \overbrace{\text{b}\text{g}\text{g}\text{g}\text{gg}\text{g}}
\end{aligned}$$

**\$SeriesShowDegree = 7; Last[\mu\_0]**

$$\begin{aligned}
& \text{CWS}[0, 0, 2 \text{CW}[\text{bgr}], \text{CW}[\text{bbgr}] - \text{CW}[\text{bgbr}] + \text{CW}[\text{bggr}] - \text{CW}[\text{bgrg}] + \text{CW}[\text{bgrr}] - \text{CW}[\text{brgr}], \\
& \frac{\text{CW}[\text{bbbgr}]}{3} - \frac{\text{CW}[\text{bbgbr}]}{2} + \frac{\text{CW}[\text{bbggr}]}{2} + \frac{\text{CW}[\text{bbgrg}]}{2} + \frac{\text{CW}[\text{bbgrr}]}{2} + \frac{\text{CW}[\text{bbrbg}]}{2} - \\
& \frac{3 \text{CW}[\text{bbgrgr}]}{2} + \frac{\text{CW}[\text{bgbr}]}{2} - \frac{3 \text{CW}[\text{bggbr}]}{2} + \frac{\text{CW}[\text{bgggr}]}{3} - \frac{\text{CW}[\text{bgrg}]}{2} + \frac{\text{CW}[\text{bgrr}]}{2} +
\end{aligned}$$

$$\begin{aligned}
& \frac{\text{CW}[bgrrg] - 3 \text{CW}[bgrrg] + \text{CW}[bgrrr] + \text{CW}[brggr] - \text{CW}[brgrr] + \text{CW}[brrgr]}{2} , \\
& \frac{\text{CW}[bbbbgr] - \text{CW}[bbbgbgr] + \text{CW}[bbbggr] - \text{CW}[bbbgrg] + \text{CW}[bbbgrr] - \text{CW}[bbbrbg]}{12} - \\
& \frac{\text{CW}[bbbrgr] + \text{CW}[bbgbbr] + \text{CW}[bbgbrr] - 3 \text{CW}[bbggbr] + \text{CW}[bbgggr] + \text{CW}[bbgggr]}{6} + \\
& \frac{\text{CW}[bbggrr] + \text{CW}[bbgrbg] - \text{CW}[bbgrgg] + \text{CW}[bbgrgr] - 5 \text{CW}[bbgrrg] + \text{CW}[bbgrrr]}{4} + \\
& \frac{3 \text{CW}[bbrbgg] + \text{CW}[bbrgbr] - 5 \text{CW}[bbrggr] + \text{CW}[bbrgrg] - 3 \text{CW}[bbrgrr]}{4} - \\
& \frac{\text{CW}[bbrrbg] + 3 \text{CW}[bbrgrg]}{4} - \text{CW}[bgbgbr] - \text{CW}[bgbgrg] + \text{CW}[bgbgrr] - \text{CW}[bgbbrb] + \\
& \frac{\text{CW}[bgbrrr] - 5 \text{CW}[bgbrr] - \text{CW}[bgbgr] + \text{CW}[bgbgr]}{6} - \\
& \frac{\text{CW}[bgbrrg] + \text{CW}[bgbgrg] - \text{CW}[bgbrrr] - 3 \text{CW}[bgbrrg] + \text{CW}[bgbrrr]}{6} - \\
& 4 \text{CW}[bgrbgr] + \text{CW}[bgrbrr] - \frac{\text{CW}[bgrggg]}{6} + \text{CW}[bgrggr] - \text{CW}[bgrgrg] + \frac{3 \text{CW}[bgrrgg]}{4} - \\
& \frac{\text{CW}[bgrrrg] + \text{CW}[bgrrrr]}{6} + \text{CW}[brbrgg] - \text{CW}[brbrgr] - \frac{\text{CW}[brgggr]}{6} + \frac{\text{CW}[brggrr]}{4} - \\
& \frac{\text{CW}[brgrgr] + \text{CW}[brgrrg]}{6} - \frac{\text{CW}[brgggr]}{4} + \frac{\text{CW}[brrgrg]}{4} - \frac{\text{CW}[brrrgr]}{6} , \\
& \frac{\text{CW}[bbbbbbgr] - \text{CW}[bbbbbgbr] + \text{CW}[bbbbbggr] + \text{CW}[bbbbbggr] + \text{CW}[bbbbbgrr] + \text{CW}[bbbrbbg]}{60} - \\
& \frac{\text{CW}[bbbbbrgr] + \text{CW}[bbbgbbr] + \text{CW}[bbbgbr] - \text{CW}[bbbggr] + \text{CW}[bbbggr] - \text{CW}[bbbggr]}{8} + \\
& \frac{\text{CW}[bbbggr] + \text{CW}[bbbgrrg] - \text{CW}[bbbgrrg] + \text{CW}[bbbgrrr] - \text{CW}[bbbrbbg] - \text{CW}[bbbrbbg]}{12} + \\
& \frac{\text{CW}[bbbrggr] - \text{CW}[bbbrgrg] - \text{CW}[bbbrgrr] + \text{CW}[bbbrrbg] + \text{CW}[bbbrgrg] + \text{CW}[bbgbgr]}{12} - \\
& \frac{\text{CW}[bbgbbr] - \text{CW}[bbgbgr] + \text{CW}[bbgbgrg] + \text{CW}[bbgbgrr] - \text{CW}[bbgbgrb] - \text{CW}[bbgbbrb]}{8} + \\
& \frac{\text{CW}[bbgbrrg] + \text{CW}[bbgbrrr] - \text{CW}[bbgbrrr] + 3 \text{CW}[bbggbb] - \text{CW}[bbggbrg]}{2} - \\
& \frac{5 \text{CW}[bbggbrr] - \text{CW}[bbgggrb] + \text{CW}[bbgggr] + \text{CW}[bbgggrg] + \text{CW}[bbgggr]}{8} + \\
& \frac{\text{CW}[bbggrbg] + \text{CW}[bbgrbr] - \text{CW}[bbggrrg] + \text{CW}[bbgrgr] - 5 \text{CW}[bbggrrg]}{2} + \\
& \frac{\text{CW}[bbggrrr] - \text{CW}[bbgrbg] - 3 \text{CW}[bbgrbgr] + \text{CW}[bbgrbgr] + \text{CW}[bbgrbrr] - \text{CW}[bbgrbg]}{12} - \\
& \frac{\text{CW}[bbgrgbr] + \text{CW}[bbgrggg] - 3 \text{CW}[bbgrgrg] + \text{CW}[bbgrgrr] + 5 \text{CW}[bbgrrgg]}{12} + \\
& \frac{\text{CW}[bbgrgrg] + \text{CW}[bbgrrrg] + \text{CW}[bbgrrrr] - \text{CW}[bbbrbbg] + \text{CW}[bbbrbbg] + \text{CW}[bbbrbgb]}{2} + \\
& \frac{\text{CW}[bbrbogg] + \text{CW}[bbrbggr] + \text{CW}[bbrbrbg] - \text{CW}[bbrbrgg] + \text{CW}[bbrbrgr] - \text{CW}[bbrbggg]}{12} -
\end{aligned}$$

$$\begin{aligned}
& 2 \text{CW}[bbrgbgr] - \frac{\text{CW}[bbrgbrr]}{2} + \text{CW}[bbrggbr] - \frac{\text{CW}[bbrgggr]}{4} + \frac{\text{CW}[bbrgggr]}{2} - \\
& \frac{5 \text{CW}[bbrggrr]}{8} + \text{CW}[bbrgrbg] + \frac{\text{CW}[bbrgrbr]}{2} - \frac{\text{CW}[bbrgrgg]}{2} + \frac{\text{CW}[bbrgrgr]}{2} + \text{CW}[bbrgrrg] - \\
& \frac{\text{CW}[bbrgrrr]}{4} + \frac{5 \text{CW}[bbrrbgg]}{8} - \frac{\text{CW}[bbrrbgr]}{2} - \frac{\text{CW}[bbrrgbg]}{2} - \frac{\text{CW}[bbrrgbr]}{2} + \\
& \frac{5 \text{CW}[bbrrggr]}{8} - \frac{\text{CW}[bbrrgrg]}{2} + \frac{3 \text{CW}[bbrrgrr]}{8} + \frac{\text{CW}[bbrrrbg]}{12} - \frac{\text{CW}[bbrrrgr]}{4} - \\
& \text{CW}[bgbgbgr] + \text{CW}[bgbgbrg] - \frac{3 \text{CW}[bgbgbrr]}{2} + \frac{\text{CW}[bgbggbr]}{2} - \frac{\text{CW}[bgbgggr]}{2} + \\
& \frac{\text{CW}[bgbggrr]}{2} - \text{CW}[bgbgrbr] + \frac{\text{CW}[bgbgrgg]}{2} - \text{CW}[bgbgrgr] + \frac{\text{CW}[bgbgrrg]}{2} + \frac{\text{CW}[bgbrbgg]}{2} + \\
& 3 \text{CW}[bgbrbgr] + \text{CW}[bgbrbrg] - \frac{\text{CW}[bgbrbrr]}{2} + \text{CW}[bgbgrg] + \text{CW}[bgbrggr] - \text{CW}[bgbrgrr] + \\
& \frac{\text{CW}[bgbrbr]}{2} - \frac{\text{CW}[bgbrrgg]}{2} - \text{CW}[bgbrrrg] + \frac{\text{CW}[bgbrrrr]}{24} + \frac{\text{CW}[bggbgrg]}{2} + \frac{\text{CW}[bggbgrr]}{2} + \\
& \frac{\text{CW}[bggbbrbr]}{2} - \frac{\text{CW}[bggbrgg]}{2} - 2 \text{CW}[bggbgr] + \text{CW}[bggbrrg] - \frac{\text{CW}[bggbrrr]}{4} + \\
& \frac{\text{CW}[bgggbrr]}{12} - \frac{\text{CW}[bggggbr]}{8} + \frac{\text{CW}[bgggggr]}{60} - \frac{\text{CW}[bgggggrg]}{24} + \frac{\text{CW}[bggggrr]}{24} + \frac{\text{CW}[bggggrrg]}{12} - \\
& \frac{\text{CW}[bgggrrg]}{4} + \frac{\text{CW}[bgggrrr]}{18} - 3 \text{CW}[bggrbgr] - \text{CW}[bggrbrg] + \text{CW}[bggrgbr] - \frac{\text{CW}[bggrggg]}{12} + \\
& \frac{\text{CW}[bggrggr]}{2} - \frac{\text{CW}[bggrgrg]}{2} + \frac{\text{CW}[bggrbr]}{2} + \frac{3 \text{CW}[bggrrgg]}{8} - \frac{\text{CW}[bggrrrg]}{12} + \\
& \frac{\text{CW}[bggrrrr]}{24} + 3 \text{CW}[bgrbgrg] - 3 \text{CW}[bgrbgrr] - \text{CW}[bgrbrbr] + 3 \text{CW}[bgrbrgr] + \\
& \text{CW}[bgrbrrg] + \text{CW}[bgrgbrr] + \text{CW}[bgrggbr] + \frac{\text{CW}[bgrgggg]}{24} - \frac{\text{CW}[bgrgggr]}{2} + \frac{\text{CW}[bgrggrr]}{2} - \\
& \text{CW}[bgrgrbr] + \frac{\text{CW}[bgrgrgg]}{2} - \text{CW}[bgrgrgr] + \frac{\text{CW}[bgrgrrg]}{2} + \text{CW}[bgrrbgr] + \frac{\text{CW}[bgrrbrr]}{2} - \\
& 2 \text{CW}[bgrrgbr] - \frac{\text{CW}[bgrrggg]}{4} - \frac{\text{CW}[bgrrggr]}{2} + \frac{\text{CW}[bgrrrgg]}{2} + \frac{\text{CW}[bgrrrgg]}{12} - \\
& \frac{\text{CW}[bgrrrrg]}{8} + \frac{\text{CW}[bgrrrrr]}{60} + \text{CW}[brbrbgr] - \text{CW}[brbrggg] - \frac{3 \text{CW}[brbrggr]}{2} + \text{CW}[brbrgrg] - \\
& \frac{\text{CW}[brbrgrr]}{2} - \frac{\text{CW}[brbrrgg]}{2} + \frac{\text{CW}[brbrrrg]}{2} + \frac{\text{CW}[brggbr]}{2} + \frac{\text{CW}[brggggr]}{24} - \frac{\text{CW}[brgggr]}{12} + \\
& \frac{\text{CW}[brgggrg]}{2} - \frac{\text{CW}[brggrrg]}{2} + \frac{\text{CW}[brggrrr]}{12} - \frac{\text{CW}[brgrbrr]}{2} - \frac{\text{CW}[brgrggr]}{2} + \text{CW}[brgrgrg] - \\
& \frac{\text{CW}[brgrgrr]}{2} - \frac{\text{CW}[brgrrgg]}{2} + \frac{\text{CW}[brgrrgr]}{2} - \frac{\text{CW}[brgrrrr]}{24} + \frac{\text{CW}[brrgggr]}{12} - \frac{\text{CW}[brrggrr]}{8} + \\
& \frac{\text{CW}[brrgrgr]}{2} - \frac{\text{CW}[brrgrrg]}{2} + \frac{\text{CW}[brrgrrr]}{12} + \frac{\text{CW}[brrrggr]}{12} - \frac{\text{CW}[brrrgrr]}{12} + \frac{\text{CW}[brrrrgr]}{24}
\end{aligned}$$