More on the previous lecture:

\[
\text{Hom}_\mathcal{O}(\mathcal{O}(\mathcal{L}, \phi), \mathcal{P}) \cong \text{Hom}_{\mathcal{L}}^{-1}(\mathcal{L}, \phi) \cong \text{Hom}_{\mathcal{O}}(\mathcal{L}, \mathcal{O}(\mathcal{L}, \phi)) \\
\text{Hom}_{\mathcal{O}}(\mathcal{O}(\mathcal{L}, \phi), \mathcal{P}) \cong \text{Tw}(\mathcal{L}, \phi) \cong \text{Hom}_{\mathcal{O}}(\mathcal{L}, \mathcal{O}(\mathcal{L}, \phi)) \\
\text{QI}(\mathcal{O}(\mathcal{L}, \phi), \mathcal{P}) \cong \text{Kos}(\mathcal{L}, \phi) \cong \text{QI}(\mathcal{L}, \phi) \\
\]

Things simplify if the operads are “quadratic”.

See video, even if this phrase appears here too often.

Yet I’m a low-faith fellow, and there’s only that much I can go without knowing that what’s being done must be done for the purpose of some good cause. In other words, I cannot work unmotivated.

And, perhaps embarrassingly, I still don’t understand the motivation for “resolutions”, even in the most basic sense.

I guess, even before “what’s a resolution”, comes “what’s a module”. I don’t even know the answer to that.

---

**Def. Gerstenhaber algebra:**

\[
\text{deg} \circ \text{commutative product},
\]

\[
A \otimes A \to A
\]
\[ (A \otimes A) \otimes (A \otimes A) \rightarrow S(A) \otimes S(A) \text{ deq } 0 \text{ bracket.} \]

(skew symmetric)

\[ \phi \]

\[ \langle \quad \rangle : A \otimes A \rightarrow A \text{ deq } 1 \text{ symmetric} \ldots \]

\[ \ldots \text{ The homology of the little disk operad.} \]

Does the little disk operad ever say something truly useful about honest braids?

"The Chevalley-Eilenberg complex of a free Lie algebra is acyclic"

Rewriting methods

Associative algebras

\[ A(V, R) = S(V) = T(V)/(x\otimes y - y \otimes x) \]

1. Choose a totally ordered basis for \( V \)

\[ V = \{ x_1 < x_2 < x_3 < \ldots \} \]

2. Consider the lexicographic order, or any other order \( \phi \) increases under concatenation

\[ x_1 < x_2 < x_3 \]

\[ x_1 x_1 < x_1 x_2 < \ldots \]

Choose a basis for \( R \) s.t.
the leading term [highest] in every relation does not appear in any other relation.

3. Interpret the relations as decreasing rewriting rules.

The "critical monomials" are those in which $X Y Z$ both $X Y$ & $Y Z$
are reducible.

4. Write the diamond for each critical monomial

```
  x_3 x_2 x_1
   \ /  \ /  \ /
  x_2 x_3 x_1  x_3 x_1 x_2
   \   \   \   
  x_2 x_1 x_3  x_1 x_3 x_2
   \   \   \   
  x_1 x_2 x_3 = x_1 x_2 x_3
```

"Confluence" — going both ways we got to the same place.

Thm: If such a procedure exists, which always leads to confluences, then the algebra is Koszul.
A basis $A$ is "words w/o leading terms"