To tell a good story about spectral sequences, it would be enough to have a good story about 5-step complexes. (3 steps for the definition of $E^r$, one more for the connecting homomorphism, one more for the homology using the connected homomorphism).

(I don’t really understand the 5-lemma).