First inputs in G.T. theory

Grothendieck 1983
Ihara Annals 1986
Drinfeld 1989
Deligne P1-3pts, MSRI 1980

Three messages:

* Large Galois reps are often faithful.
* 2-level principle
* The theory is or should be “non-linear”

Belyi (1976) Any smooth curve \( X \) over \( \overline{\mathbb{Q}} \) can be realized as a covering of \( \mathbb{P}^1 \) ramified over at most 0, 1, \( \infty \).

\([\text{SGA1}]: \quad X \text{ geometric connected scheme over } (k, \kappa), \]

\[ \mathbb{T}_i^{\text{geom}}(X) = \mathbb{T}_i(X \otimes \kappa), \quad \text{then} \]

\[ 1 \to \mathbb{T}_i^{\text{geom}}(X) \to \mathbb{T}_i(X) \to \text{Gal}(\kappa) \to 1 \]

is exact.

With \( X = \mathbb{P}^*: = \mathbb{P}^1 \setminus \{0, 1, \infty\} \) get
\[ G_k \rightarrow \text{Out}(\text{TT}_1 \alpha_{\text{mon}}(X)) \]

\[ G_\Omega \rightarrow \text{Out}(\text{TT}_1(\Omega^*)) \]

\[ \hat{\Omega} \leftarrow \text{profinite completion}. \]

Rest in video: How \( \sigma_k \) acts on \( \hat{\text{TT}}_1(\Omega^*) \).

Explicit constraints on the image of \( G_\Omega \) in \( \text{Out}^*(\hat{\Omega}) \) — this is the \( \text{AT} \) group.