NCGE Talk post-Mortem
January 24-13 7:05 AM

The Problem. Let $G = \langle y_1, \ldots, y_n \rangle$ be a subgroup of $S_n$, with $n = O(100)$. Before you die, understand $G$:
2. Given $\sigma \in S_n$, decide if $\sigma \in G$.
3. Write a $\sigma \in G$ in terms of $y_1, \ldots, y_n$.
4. Produce random elements of $G$.

The Complementary Analog. Let $V = \text{span}(v_1, \ldots, v_n)$ be a subspace of $\mathbb{R}^n$. Before you die, understand $V$.

Solution: Gaussian Elimination. Prepare an empty table,

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & \cdots & n-1 & n \\
\end{array}
\]

for $v_1, \ldots, v_n$ in order. To feed a non-zero $v_i$ find its pivotal position $i$.
1. If box $i$ is empty, put $v_i$ there.
2. If box $i$ is occupied, find a combination $\delta$ of $v$ and $i$, that eliminates the pivot, and feed $\delta$.

Non-Commutative Gaussian Elimination
Prepare a mostly-empty table,

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & \cdots & n-1 & n \\
\end{array}
\]

Feed $v_1, \ldots, v_n$ in order. To feed a non-identity $\sigma$, find its pivotal position $i$ and let $j := \sigma(i)$.
1. If box $(i, j)$ is empty, put $\sigma$ there.
2. If box $(i, j)$ contains $\sigma$, feed $\sigma' := \sigma^{-1}\sigma$.

The Twist. When done, for every occupied $(i, j)$ and $(k, l)$, feed $\mu_i \mu_j \mu_k \mu_l$. Repeat until the table stops changing.

Claim 1. The process stops in our lifetimes, after at most $O(n^2)$ operations. Call the resulting table $T$.

Claim 2. Every $\sigma_i \in T$ in $G$.

Claim 3. Anything fed in $T$ is now a monotone product in $T$:

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & \cdots & n-1 & n \\
\end{array}
\]

Homework Problem 1. Can you do cozes?

A Demo Program

\[\begin{array}{cccccc}
1 & 2 & 3 & 4 & \cdots & n-1 & n \\
\end{array}\]

Change to match with program

Swap.

The Results

Table: [Feed[1], \{1 \rightarrow Count[Range[n]]\}, \{1 \rightarrow Feed[1] \rightarrow Cycle}\}, \{1, 0, 6\}]

In limit:

\[\begin{array}{cccccc}
1 & 2 & 3 & 4 & \cdots & n-1 & n \\
\end{array}\]