Operads in \((\text{Top}, x)\) or in \((\text{sSets}, x)\)

If \(P\) is an operad a \(P\)-algebra is

\[
(\star) \quad P(n) \times A^n \to A \quad \left[\text{satisfying some conditions}\right]
\]

If \(Q\) operad morphism make every \(P\)-algebra into a \(Q\)-algebra.

For us operads are non-symmetric.

If we forget the equations in \((\star)\), the resulting algebras are "controlled" by \(\text{Free}(P)\), whose definition is obvious, using finite, rooted, planar trees:

\[
E \in \text{Free}(P)(4)
\]

Operad structure by "grafting".

There is also \(\text{Free}_{\leq}(P)\) which is

\[
\text{Free}(P) / \text{Idem} = |
\]

There are maps of operads
\[ \text{Free}(P) \xrightarrow{e} \text{Free}_{\Pi}(P) \xrightarrow{\rho < e} W_{\Pi}(P) \]

So every \( P \)-algebra is also a \( \text{Free}(P) \)-algebra.

"Controls the equation up to homotopy": \( W_{\Pi}(P) = \text{The Boardman-Vogt resolution} \)

Elements of \( W_{\Pi}(P) \) are like elements of \( \text{Free}(P) \), with edges labeled with lengths in \([0, 1]\).

\[ \text{edges of length } 0 \text{ can be contracted.} \]

\[ \text{all non-internal edges are labeled } 1 \]

\[ \text{a homotopy between...} \]

A \( W_{\Pi}(P) \) algebra is like a \( P \)-algebra, except eqns's only hold up to homotopy.

Example: \( P = \text{ASS} \) (assoc. algebra w/n unit)
$W(\text{Ass})(1) = pt.$

$W(\text{Ass})(2) = \gamma$

$W(\text{Ass}(3)) = \xrightarrow{\gamma_t} \xrightarrow{\gamma} \xrightarrow{\gamma_t}$

a "subdivided homotopy"

$W(\text{Ass}(4)) =$

More in video.