Extremely brief review of "calculus of functors"

Review of Embedding calculus.

Connection to knots, links etc: The good, the bad, the ugly.

Calculus of functors ends up playing a relatively minor role.

Based on work of Ismar Volić & Brian Munson.

Calculis of functors:

\[ F : C \to \text{Top} \quad (w/ \text{nice properties}) \]

Construct "approximations" \( T_k F : C \to \text{Top} \)

W/ nat. trans.

\[ F \leftarrow T_k F \quad \text{co-cartesian} \]

\[ \exists \quad \text{homotopy} \]

\[ T_k F \quad \text{"sees } F \text{ up to } K\text{-cubics"} \]

Strongly homotopy co-cartesian

Homotopy cartesian
Ideal: 1. $\text{holim} \ T_kF$ should converge in the sense that homotopy groups should stabilize.

2. $\text{holim} \ T_kF$ should be weakly equiv. to $F$.

Manifold calculus — Embeddings.

Source model $M^m$, target model $N^n$

Use $\mathcal{O}(M)^0$: open subsets of $M$, morphisms are "reverse inclusions".

$F$ is $\text{Emb}(-, N): \mathcal{O}(M)^0 \to \text{spaces}.$

Nice properties:

1. $U \simeq_N V$ isotopy $\Rightarrow$ homotopy equiv. $\text{Emb}(U, N) \simeq \text{Emb}(V, N)$

2. If $U = U_i'$ with $U'_i \subset U_{i+1}$, then $\text{Emb}(U, N) = \text{holim} \text{Emb}(U_i', N)$

Convergence result: The functor $U \to \text{Emb}(U, N)$ is $(N-2)$-analytic with excess 3-$n$.

$\text{Emb}(M, N) \to T_k \text{Emb}(M, N)$ is $(k(n-M-2)+1-n)$-connected. So if $n-M-2 > 0$ then the above is
a weak equivalence.

Applications to knots KC.

Knot space: \( \text{Emb}(S^1, N^3) \)

Link space: \( \text{Emb}(L^1 S^1, N^3) \)

Formulas give no information about convergence!

We want to study \( T_0(\text{knot space}) \) by studying

\[ H^0(\text{knot space}, A) \]

We have \( H^0(T_k \text{ knot}) \rightarrow H^0(\text{knot}) \).

How good a source of invariants is this?

\( H^0(T_k \text{ knot}) \) “are” F.T. invariants.

**Step 1** Build a model for \( T_k \text{ knot} \) — simplicial, pieces are configuration spaces of points and directions in \( \mathbb{R}^3 \).

**Step 2** Compactify.

**Step 3** Apply big machinery (spectral sequences)

**Step 4** Analyze the SS & show convergence results; identify the limits.